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LEONARD G. JOHNSON
EDITOR

WANG H. YEE
DIRECTOR

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RANKING THEORY AND EVIDENCE IN COMPLIANCE RELIABILITY

THE LOGARITHMIC PARAMETRIC CONFIDENCE FORMULA FOR UPPER STANDARD TESTS

Let b = Weibull Slope

Let $\rho_N = \left(\dfrac{X_N}{X_o}\right)$ (X_o = Standard)

(X_N = largest of N observations)

THEN :

$$1 - (1 - C) \text{ Rank of } X_N = 1 - R(X_N) = \left[1 - R(X_o)\right]^{\rho_N^b}$$

The parametric (1 - C) Rank of the Nth in N is

$$1 - \left(1 - \frac{N - .3}{N + .4}\right)^{e^{\frac{t_{1-C}}{\sqrt{N}}}}$$

$$\therefore \quad \left(1 - \frac{N - .3}{N + .4}\right)^{e^{\frac{t_{1-C}}{\sqrt{N}}}} = \left[1 - R(X_o)\right]^{\rho_N^b}$$

OR

$$\left(\frac{.7}{N + .4}\right)^{e^{\frac{t_{1-C}}{\sqrt{N}}}} = \left[1 - R(X_o)\right]^{\rho_N^b}$$

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$$e^{\frac{t_{1-C}}{\sqrt{N}}} \ln\left(\frac{.7}{N + .4}\right) = \rho_N^{\,b} \ln\left[1 - R(X_o)\right]$$

$$e^{\frac{t_{1-C}}{\sqrt{N}}} = \frac{\rho_N^{\,b} \ln\left[1 - R(X_o)\right]}{\ln\left(\frac{.7}{N + .4}\right)}$$

$$\frac{t_{1-C}}{\sqrt{N}} = b \ln \rho_N + \ln\ln\left[1 - R(X_o)\right] - \ln\ln\left(\frac{.7}{N + .4}\right)$$

$$t_{1-C} = \sqrt{N}\left\{ b \ln \rho_N + \ln\ln\left[1 - R(X_o)\right] - \ln\ln\left(\frac{.7}{N + .4}\right)\right\}$$

$$= \sqrt{N}\left\{ b \ln \rho_N + \ln\ln\left[\frac{1}{1 - R(X_o)}\right] - \ln\ln\left(\frac{N + .4}{.7}\right)\right\}$$

$$C = \frac{1}{1 + e^{+1.8138\, t_{1-C}}}$$

FOR THE SEVEN VEHICLE CASE IN VOL. 6, BULLETIN 3:

$N = 7$; $b = 6.76$; $\rho_N = .80667$

$R(X_o) = .999$ (DESIRED RELIABILITY)

HENCE,

$$t_{1-C} = \sqrt{7}\left\{6.76 \ln .80667 + \ln\ln\left(\frac{1}{.001}\right) - \ln\ln\left(\frac{7.4}{.7}\right)\right\}$$

$$= -.99892 \qquad C = \frac{1}{1 + e^{-1.81185}} = .8596$$

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$$e^{\frac{t_{1-C}}{\sqrt{N}}} = \frac{\rho_N^b \ln\left[1 - R(X_o)\right]}{\ln\left(\frac{.7}{N + .4}\right)}$$

$$e^{1.8138\, t_{1-C}} = \left[\frac{\rho_N^b \ln\left[1 - R(X_o)\right]}{\ln\left(\frac{.7}{N + .4}\right)}\right]^{1.8138\sqrt{N}}$$

$$\therefore \quad C = \frac{1}{1 + \left[\frac{\rho_N^b \ln\left[1 - R(X_o)\right]}{\ln\left(\frac{.7}{N + .4}\right)}\right]^{1.8138\sqrt{N}}}$$

FOR THE SEVEN VEHICLE EXAMPLE :

$$\rho_N^b = .23402 \qquad \ln\left[1 - R(X_o)\right] = -6.90776$$

$$N = 7 \qquad \ln\left(\frac{.7}{N + .4}\right) = -2.35815$$

$$C = \frac{1}{1 + \left[\frac{.23402\,(-6.90776)}{-2.35815}\right]^{1.8138\sqrt{7}}} = \underline{\underline{.8596}}$$

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LOWER LIMIT ASSURANCE TESTING

X_o = Standard (Which must be exceeded)

X_1 = Smallest of N observations

$$\rho_1 = \left(\frac{X_1}{X_o}\right) > 1$$

b = Weibull Slope

$R(X_o)$ = Desired Reliability to Standard.

NON-PARAMETRIC CONFIDENCE

$$C = 1 - \left[R(X_o)\right]^{N \rho_1^{b}}$$

LOGARITHMIC PARAMETRIC CONFIDENCE

$$C = 1 \div \left\{ 1 + \left[\frac{\ln\left(\frac{N - .3}{N + .4}\right)}{\rho_1^{b} \ln R(X_o)} \right]^{1.8138 \sqrt{N}} \right\}$$

FORMULA FOR LOGARITHMIC PARAMETRIC EVIDENCE

UPPER LIMIT ASSURANCE TESTING

$$E = -1.8138 \sqrt{N} \left\{ \ln \ln \frac{1}{1 - R(X_N)} - \ln \ln \left(\frac{N + .4}{.7} \right) \right\}$$

with $R(X_N) = 1 - \left[1 - R(X_o)\right]^{\rho_N^{b} N}$; X_o = Standard

X_N = Largest of N ; b = Weibull Slope ; $\rho_N = \left(\frac{X_N}{X_o}\right) < 1$

$R(X_o)$ = Desired Reliability to Standard X_o .

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LOWER LIMIT ASSURANCE TESTING

$$E = 1.8138\sqrt{N}\left\{\ln\ln\frac{1}{R(X_1)} - \ln\ln\left(\frac{N+.4}{N-.3}\right)\right\}$$

with $R(X_1) = \left[R(X_o)\right]^{\rho_1^{\,b}}$; $\rho_1 = \left(\frac{X_1}{X_o}\right) > 1$

X_o = Standard ; X_1 = Smallest of N ; b = Weibull Slope

$R(X_o)$ = Desired Reliability to Standard X_o .

THREE TYPES OF RANKS

CONSIDER THE FIRST OF N (i.e., the lowest value in a sample of N)

NON-PARAMETRIC RANK

Non-Parametric C-Rank of First of $N = 1 - (1 - C)^{1/N}$

ARITHMETIC PARAMETRIC RANK

Arithmetric Parametric C-Rank of First of $N = 1 - \left(\frac{N - .3}{N + .4}\right)^{\frac{1}{1 - \frac{Z_c}{\sqrt{N}}}}$

where Z_c = Normal Z-Score for cumulative area c .

LOGARITHMIC PARAMETRIC RANK

Logarithmic Parametric C-Rank of First of $N = 1 - \left(\frac{N - .3}{N + .4}\right)^{e^{\frac{Z_c}{\sqrt{N}}}}$

where Z_c = Normal Z-Score for cumulative area c.

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THREE TYPES OF RANKS

CONSIDER THE LAST OF N : (i. e., The largest value in a sample of N)

NON-PARAMETRIC C-RANK

Non-Parametric C-Rank of the last of N = $C^{1/N}$

ARITHMETIC PARAMETRIC C-RANK

$$\text{Arithmetic Parametric C-Rank of last of N} = 1 - \left(\frac{.7}{N + .4}\right)^{\frac{1}{1 - \frac{Z_c}{\sqrt{N}}}}$$

Z_c = Normal Z-Score for cumulative area c.

LOGARITHMIC PARAMETRIC C-RANK

$$\text{Logarithmic Parametric C-Rank of Last of N} = 1 - \left(\frac{.7}{N + .4}\right)^{e^{\frac{Z_c}{\sqrt{N}}}}$$

RELIABILITY

LOWER LIMIT ASSURANCE TESTING

NON-PARAMETRIC RELIABILITY WITH CONF. C :

$$R_C(X_o) = (1 - C)^{1/N\rho_1^{\,b}}$$

where

N = Sample Size ; b = Weibull Slope ; $\rho_1 = \left(\frac{X_1}{X_o}\right) > 1$

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ARITHMETIC PARAMETRIC RELIABILITY WITH CONF. C

$$R_C(X_o) = \left(\frac{N - .3}{N + .4}\right)^{\frac{1}{\rho_1^b\left(1 - \frac{Z_c}{\sqrt{N}}\right)}}$$

where Z_c = Normal Z-Score to cumulative area c .

LOGARITHMIC PARAMETRIC RELIABILITY WITH CONF. C

$$R_C(X_o) = \left(\frac{N - .3}{N + .4}\right)^{\frac{1}{\rho_1^b} e^{\frac{Z_c}{\sqrt{N}}}}$$

RELIABILITY

UPPER LIMIT ASSURANCE TESTING

NON-PARAMETRIC RELIABILITY WITH CONF. C

$$R_C(X_o) = 1 - \left[1 - (1 - C)^{1/N}\right]^{1/\rho_N^b}$$

where N = Sample Size $(X_1, X_2, \ldots X_N)$ all less than X_o .

$\rho_N = \left(\frac{X_N}{X_o}\right) < 1$; b = Weibull Slope

ARITHMETIC PARAMETRIC RELIABILITY WITH CONF. C

$$R_C(X_o) = 1 - \left(\frac{.7}{N + .4}\right)^{\frac{1}{\rho_N^b\left(1 + \frac{Z_c}{\sqrt{N}}\right)}}$$

where Z_c = Normal Z-Score for cumulative area c.

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LOGARITHMIC PARAMETRIC RELIABILITY WITH CONF. C

$$R_C(X_o) = 1 - \left(\frac{.7}{N + .4}\right)^{\left(1/\rho_N^b\right) e^{-\frac{Z_c}{\sqrt{N}}}}$$

- -

EVIDENCE

LOWER LIMIT ASSURANCE TESTING

NON-PARAMETRIC EVIDENCE :

$$E = \ln\left(R^{-N\rho_1^b} - 1\right)$$

where $(X_1, X_2, \ldots X_N)$ all above X_o

R = Desired Reliability to X_o ; b = Weibull Slope

N = Sample Size ; $\rho_1 = \left(\frac{X_1}{X_o}\right) > 1$

ARITHMETIC PARAMETRIC EVIDENCE :

$$E = 1.8138\sqrt{N}\left[1 - \frac{\ln\left(\frac{N - .3}{N + .4}\right)}{\rho_1^b \ln R}\right]$$

LOGARITHMIC PARAMETRIC EVIDENCE :

$$E = 1.8138\sqrt{N}\left\{\ln\ln\frac{1}{R} + b\ln\rho_1 - \ln\ln\left(\frac{N + .4}{N - .3}\right)\right\}$$

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EVIDENCE

UPPER LIMIT ASSURANCE TESTING

NON-PARAMETRIC EVIDENCE :

$$E = \ln\left\{\left[1 - (1 - R)\,\rho_N^{\,b}\right]^{-N} - 1\right\}$$

where

R = Desired Reliability to Upper Specification X_o .

b = Weibull Slope ; N = Sample Size : $(X_1, X_2, \ldots X_N)$ all below X_o

$$\rho_N = \left(\frac{X_N}{X_o}\right) < 1$$

ARITHMETIC PARAMETRIC EVIDENCE :

$$E = 1.8138\sqrt{N}\left[\frac{\ln\left(\frac{.7}{N + .4}\right)}{\rho_N^{\,b}\,\ln(1 - R)} - 1\right]$$

LOGARITHMIC PARAMETRIC EVIDENCE :

$$E = 1.8138\sqrt{N}\left\{\ln\ln\left(\frac{N + .4}{.7}\right) - \ln\ln\left(\frac{1}{1 - R}\right) - b\ln\rho_N\right\}$$

EVIDENCE REQUIRED IN COMPLIANCE TESTING

FOR A BREAK-EVEN POLICY :

$$E_{REQ.} = \ln\left(\frac{\text{DOLLAR LOSS DUE TO NON-COMPLIANCE}}{\text{NET PROFIT WHEN IN COMPLIANCE}}\right)$$

FOR A (K TO 1) ODDS AGAINST DOLLAR LOSSES :

$$E_{REQ.} = \ln\left(\frac{K \times \text{DOLLAR LOSS DUE TO NON-COMPLIANCE}}{\text{NET PROFIT WHEN IN COMPLIANCE}}\right)$$

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EXAMPLE OF LOWER LIMIT COMPLIANCE

A certain manufacturer's machine is required to be guaranteed to operate failure free for 1000 hours. Five such machines are tested under service conditions with the following hours to failure :

MACHINE NUMBER	HOURS TO FAILURE
1	3405 hours
2	4002 hours
3	2576 hours
4	3040 hours
5	3695 hours

The predicted net profit when in compliance is $11,000,000 from 1000 such machines to be produced and sold , while the estimated dollar loss in case of non-compliance is $20,000,000 .

If 10 to 1 odds are desired for long term gains, does the above data set on five machines constitute sufficient evidence of compliance to the 1000 hour failure-free warranty ?

SOLUTION

From the data		plotted at Median ranks		we obtain
	2576 hrs.		.129	
	3040 hrs.		.315	
	3405 hrs.		.500	
	3695 hrs.		.685	
	4002 hrs.		.871	

Weibull Parameters $\left\{ b = 6.02245,\ \theta = 3586 \text{ hrs.} \right\}$.

$(X_o = 1000 \text{ hrs.})$

$$\rho_1 = \frac{X_1}{X_o} = \frac{2576}{1000} = 2.576$$

For 1000 machines to be produced and sold, the Required Reliability is

$$\frac{1000}{1001} = .999$$

(USE LOGARITHMIC PARAMETRIC EVIDENCE)

The Accumulated Logarithmic Parametric Evidence is

$$E_{\log.\,par.} = 1.8138\sqrt{5}\left[\ln\ln\left(\frac{1}{.999}\right) + 6.02245 \ln 2.576 - \ln\ln\left(\frac{5.4}{9.7}\right)\right] = 3.106 \text{ Units of Evidence}$$

FROM THE DOLLAR FIGURES AND 10 TO 1 ODDS :

$$E_{req.} = \ln\left(\frac{10 \times 20,000,000}{11,000,000}\right) = 2.900 \text{ Units of Evidence}$$

CONCLUSION : The test on 5 machines provided sufficient evidence of compliance .

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