Statistical Bulletin
Reliability & Variation Research

DETROIT RESEARCH INSTITUTE
21900 GREENFIELD ROAD • OAK PARK, MICHIGAN 48237 • (313) 968-1818

LEONARD G. JOHNSON
EDITOR

WANG H. YEE
DIRECTOR

Volume 6 Bulletin 2 May, 1976

WEIBULL FIELD REPAIR THEORY

Published 8 times a year by the Detroit Research Institute. Subscription Rate: One year $30.00. Quantity rates on request.
Address subscription orders and editorial correspondence to:
Detroit Research Insitute, 21900 Greenfield Road, Oak Park, Michigan 48237

I: THE RELATION BETWEEN WEIBULL LIFE AND ENTROPY

A three-parameter Weibull distribution of life X has the cumulative distribution function F(X) defined by the formula

$$F(X) = 1 - e^{-\left(\frac{X - \alpha}{\theta - \alpha}\right)^b}$$

where X = life (variable)

α = minimum life

θ = characteristic life

b = Weibull slope (shape parameter)

F(X) = fraction of the population failed in time X

In particular: $F(\alpha) = 0$

$$F(\theta) = 1 - e^{-1} = .632$$

Furthermore: $$B_Q \text{ Life} = \alpha + (\theta - \alpha)\left(\ln \frac{1}{1 - Q}\right)^{1/b}$$

$$\text{Median Life } (Q=.5) = \alpha + (\theta - \alpha)(\ln 2)^{1/b}$$

$$\text{Mean Life} = \alpha + (\theta - \alpha)\,\Gamma(1 + 1/b)$$

ENTROPY at time X is defined as the natural logarithm of the reciprocal of the fraction survived at X. Thus, if we denote the entropy at X by the symbol $\mathcal{E}(X)$, it follows that this entropy has the formula

$$\mathcal{E}(X) = \ln \frac{1}{1 - F(X)}$$

Since $F(X) = 1 - e^{-\left(\frac{X-\alpha}{\theta-\alpha}\right)^b}$, we have that

$$1 - F(X) = e^{-\left(\frac{X-\alpha}{\theta-\alpha}\right)^b}$$

or $$\frac{1}{1 - F(X)} = e^{+\left(\frac{X-\alpha}{\theta-\alpha}\right)^b}$$

Hence, for a three-parameter Weibull, the ENTROPY FUNCTION Has the formula

$$\mathcal{E}(X) = \left(\frac{X-\alpha}{\theta-\alpha}\right)^b$$

II : CUMULATIVE FAILURE PROBABILITY IN TERMS OF ENTROPY

As a function of ENTROPY $\mathcal{E}$, the cumulative probability of failure is

$$F = 1 - e^{-\mathcal{E}}$$

This is an EXPONENTIAL DISTRIBUTION of ENTROPY having a MEAN VALUE of UNITY. Since an exponential distribution has no memory, we can write the following theorem:

<u>THEOREM 1 :</u> The MEAN ENTROPY between consecutive failures on the same machine is <u>unity</u> .

III : ESTIMATING THE NEEDED FIELD REPAIRS ON A MACHINE IN ELAPSED TIME X

Let r = Ave. number of repairs in elapsed time X

The accumulated entropy in elapsed time X on a machine with Weibull parameters (α, θ, b) is

$$\mathcal{E}(X) = \left(\frac{X-\alpha}{\theta-\alpha}\right)^b$$

According to THEOREM 1, the MEAN ENTROPY between consecutive failures is UNITY. Hence, for r repairs in elapsed time X, it follows that

$$\frac{\mathcal{E}(X)}{r} = 1$$

$$r = \mathcal{E}(X) = \left(\frac{X-\alpha}{\theta-\alpha}\right)^b$$

Thus, the average number of repairs to be expected in elapsed time X is equal to the ENTROPY at X. Actually, we are dealing with the SUM OF r ENTROPIES TO FAILURE, averaging out to a MEAN VALUE of a UNIT of ADDITIONAL ENTROPY PER FAILURE.

IV: THE THEORETICAL DISTRIBUTION OF THE REPAIR TOTAL PER MACHINE

Since each repair represents an entropy, it follows that we are dealing with a SUM OF r ENTROPIES, where each entropy comes from an exponential distribution whose mean value is unity.

It is well known that a random sum of exponentially distributed items whose mean is unity would have the following statistical parameters:

$$\left\{\begin{array}{lcl} \text{MEAN} & = & r \\ \text{STANDARD DEVIATION} & = & \sqrt{r} \\ \text{SKEWNESS} & = & \dfrac{2}{\sqrt{r}} \end{array}\right\}$$

Thus, we can completely describe tha distribution of repairs needed on a machine in elapsed time X by a PEARSON TYPE III DISTRIBUTION with these three parameters, where

$$\left\{\begin{array}{lcl} \text{MEAN NO. OF REPAIRS} = r & = & \left(\dfrac{X-\alpha}{\theta-\alpha}\right)^{b} \\ \text{STANDARD DEVIATION (of No. of repairs)} = \sqrt{r} & = & \sqrt{\left(\dfrac{X-\alpha}{\theta-\alpha}\right)^{b}} \\ \text{SKEWNESS(of Distr. of repairs)} = \dfrac{2}{\sqrt{r}} & = & \dfrac{2}{\sqrt{\left(\dfrac{X-\alpha}{\theta-\alpha}\right)^{b}}} \end{array}\right\}$$

V : A NUMERICAL EXAMPLE

PROBLEM:

A certain machine has Weibull parameters

$$\left\{ \begin{array}{lllll} \text{MINIMUM LIFE} & = \alpha & = & 40 & \text{hours} \\ \text{CHARACTERISTIC LIFE} & = \theta & = & 280 & \text{hours} \\ \text{WEIBULL SLOPE} & = b & = & 1.25 & \end{array} \right\}$$

Estimate the repairs required in 1000 hours by finding an interval (r_L to r_U) repairs which would represent a 90% confidence interval.

SOLUTION :

The entropy at 1000 hours is

$$\mathcal{E}(1000) = \left(\frac{1000 - 40}{280 - 40}\right)^{1.25} = 5.65686$$

Thus, the AVE. NO. OF REPAIRS in 1000 hours is $r = 5.65686$

The STANDARD DEVIATION of the No. of repairs in 1000 hours is

$$\sigma = \sqrt{r} = \sqrt{5.65686} = 2.37841$$

The SKEWNESS of the distribution of the no. of repairs is

$$\alpha_3 = \frac{2}{\sqrt{r}} = \frac{2}{2.37841} = 0.84$$

The lower 5% t-score for skewness 0.84 is $t_{.05} = -1.374$ (See APPENDIX)

The upper 95% t-score for skewness 0.84 is $t_{.95} = +1.847$ (See APPENDIX)

Thus, the LOWER LIMIT of the 90% confidence interval on the required number of repairs in 1000 hours is

$$r_L = r + t_{.05}\sigma = 5.65686 - 1.374(2.37841) = 2.38892$$

The UPPER LIMIT of the same 90% confidence band is

$$r_U = r + t_{.95}\sigma = 5.65686 + 1.847(2.37841) = 10.04978$$

Thus, with 90% confidence, 2 to 10 repairs will be needed in the first 1000 hours of operation.

For a total of N such machines in the field for 1000 hours each, the repair total would be between 2.38892 N and 10.04978 N with 90 % confidence.

DRI STATISTICAL BULLETIN
Vol. 6, Bulletin 2
May , 1976

APPENDIX

Table of t-Scores for Pearson Type III Quantile Levels

Skewness	$t_{.01}$	$t_{.05}$	$t_{.10}$	$t_{.20}$	$t_{.30}$	$t_{.40}$	$t_{.50}$	$t_{.60}$	$t_{.70}$	$t_{.80}$	$t_{.90}$	$t_{.95}$	$t_{.99}$
-2.0	-3.605	-1.996	-1.303	-.609	-.204	.084	.307	.489	.643	.777	.895	.949	.990
-1.8	-3.500	-1.982	-1.318	-.643	-.241	.050	.281	.475	.645	.799	.945	1.020	1.087
-1.6	-3.390	-1.962	-1.329	-.676	-.278	.016	.254	.459	.643	.817	.995	1.093	1.197
-1.4	-3.270	-1.938	-1.337	-.705	-.316	-.018	.225	.440	.638	.832	1.041	1.166	1.317
-1.2	-3.150	-1.910	-1.341	-.737	-.348	-.053	.195	.418	.629	.843	1.086	1.243	1.449
-1.0	-3.023	-1.877	-1.340	-.758	-.381	-.088	.166	.390	.618	.851	1.127	1.317	1.589
- .8	-2.890	-1.839	-1.336	-.780	-.417	-.122	.132	.369	.604	.866	1.166	1.388	1.732
- .6	-2.755	-1.797	-1.329	-.800	-.444	-.156	.099	.342	.588	.857	1.200	1.458	1.880
- .4	-2.620	-1.750	-1.317	-.816	-.473	-.189	.066	.314	.568	.855	1.231	1.528	2.030
- .2	-2.472	-1.700	-1.304	-.830	-.500	-.238	.034	.284	.548	.850	1.258	1.586	2.180
0	-2.326	-1.645	-1.282	-.840	-.524	-.254	0	.254	.524	.840	1.282	1.645	2.326
.2	-2.180	-1.586	-1.258	-.850	-.548	-.284	-.034	.238	.500	.830	1.304	1.700	2.472
.4	-2.030	-1.528	-1.231	-.855	-.568	-.314	-.066	.189	.473	.816	1.317	1.750	2.620
.6	-1.880	-1.458	-1.200	-.857	-.588	-.342	-.099	.156	.444	.800	1.329	1.797	2.765
.8	-1.732	-1.388	-1.166	-.866	-.604	-.369	-.132	.122	.417	.780	1.336	1.839	2.890
1.0	-1.589	-1.317	-1.127	-.851	-.618	-.394	-.166	.088	.381	.758	1.340	1.877	3.023
1.2	-1.449	-1.243	-1.086	-.843	-.629	-.418	-.195	.053	.348	.737	1.341	1.910	3.150
1.4	-1.317	-1.166	-1.041	-.832	-.638	-.440	-.225	.018	.316	.705	1.337	1.938	3.270
1.6	-1.197	-1.093	- .995	-.817	-.643	-.459	-.254	-.016	.278	.676	1.329	1.962	3.390
1.8	-1.087	-1.020	- .945	-.799	-.645	-.475	-.281	-.050	.241	.643	1.318	1.982	3.500
2.0	- .990	- .949	- .895	-.777	-.643	-.489	-.307	-.084	.204	.609	1.303	1.996	3.605