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DEFINITIONS AND PRINCIPLES FOR STOCK MARKET TIMING

DEFINITION OF NET BUYING VOLUME

Let us go from time t through time $t + \eta$

Let S = Total No. of shares outstanding

Let X_t = Price at time t

Let $X_{t+\eta}$ = Price at time $t + \eta$

Let $F(X)$ = CDF (Cumulative Distribution Function) of Supply
(This is also known as the "Supply Curve".)

Let $f(X)$ = PDF (Probability Density Function) of Supply
(This is the derivative of $F(X)$ with respect to X.)

Let NBV = NET BUYING VOLUME (From time t throught time $t + \eta$)

Then , mathematically,

$$NBV = S\left[F(X_{t+\eta}) - F(X_t)\right]$$

or ,

$$NBV = S\int_{X_t}^{X_{t+\eta}} f(X)\,dX$$

$$= S(X_{t+\eta} - X_t) \cdot (\text{Ave. } f \text{ in interval } X_t \text{ to } X_{t+\eta})$$

Thus , $F(X_{t+\eta}) - F(X_t) = \frac{NBV}{S}$

$$F(X_{t+\eta}) = F(X_t) + \frac{NBV}{S}$$

$$X_{t+\eta} = F^{-1}\left[F(X_t) + \frac{NBV}{S}\right]$$

INTERRELATIONSHIPS BETWEEN MARKET VALUES AND NET BUYING VOLUME

Take the equation $F(X_{t+\eta}) - F(X_t) = \frac{NBS}{S}$, and divide both sides by $X_{t+\eta} - X_t$:

$$\frac{F(X_{t+\eta}) - F(X_t)}{X_{t+\eta} - X_t} = \frac{NBV}{S(X_{t+\eta} - X_t)}$$

According to the MEAN VALUE THEOREM for continuous functions, the left side of this last equation is equal to the derivative of $F(X)$ evaluated at some X between X_t and $X_{t+\eta}$, i. e., it is equal to $f(X_t + \xi\eta)$ where ξ is a positive number between 0 and 1 .

Therefore ,

$$\frac{NBV}{X_{t+\eta} - X_t} = S\ f(X_{t+\xi\eta}) = S\ f_{ave.}$$

or ,

$$X_{t+\eta} - X_t = \frac{NBV}{S\ f_{ave.}}$$

In words, this last relation says that

$$\text{Change in Price} = \frac{\text{Net Buying Volume}}{(\text{Total Shares Outstanding})\begin{pmatrix}\text{Ave. PDF Ordinate of}\\ \text{Market Value on the}\\ \text{Supply Curve}\end{pmatrix}}$$

Also , as a result of this relation we can draw the following conclusions :

(1) For a given volume of business, the price changes are more volatile for those securities with small S and small f_{ave}.

(2) A small $f_{ave.}$ indicates the stock is undervalued, since the market price is (in such a case) far below the middle of the supply curve.

(3) A stock is undervalued when the market price is at such a location on the supply curve so as to make $S\,f_{ave.}$ unusually small.

Mathematically, we can write

$$S\,f_{ave.} = \frac{\Delta V}{\Delta X}$$

where
V = Volume
X = Price

THE THEORETICAL EFFECT OF EARNINGS

Let E = Earnings

Then , $X_{t+\eta} - X_t = \dfrac{K\,E}{S}$ (K = a constant)

Therefore , $\dfrac{K\,E}{S} = \dfrac{NBV}{S\,f_{ave.}}$

Thus , $f_{ave.} = \dfrac{NBV}{K\,E}$

In words , this says that

$f_{ave.}$ is proportional to $\left(\dfrac{\text{Volume in any Period}}{\text{Earnings in the same Period}} \right)$

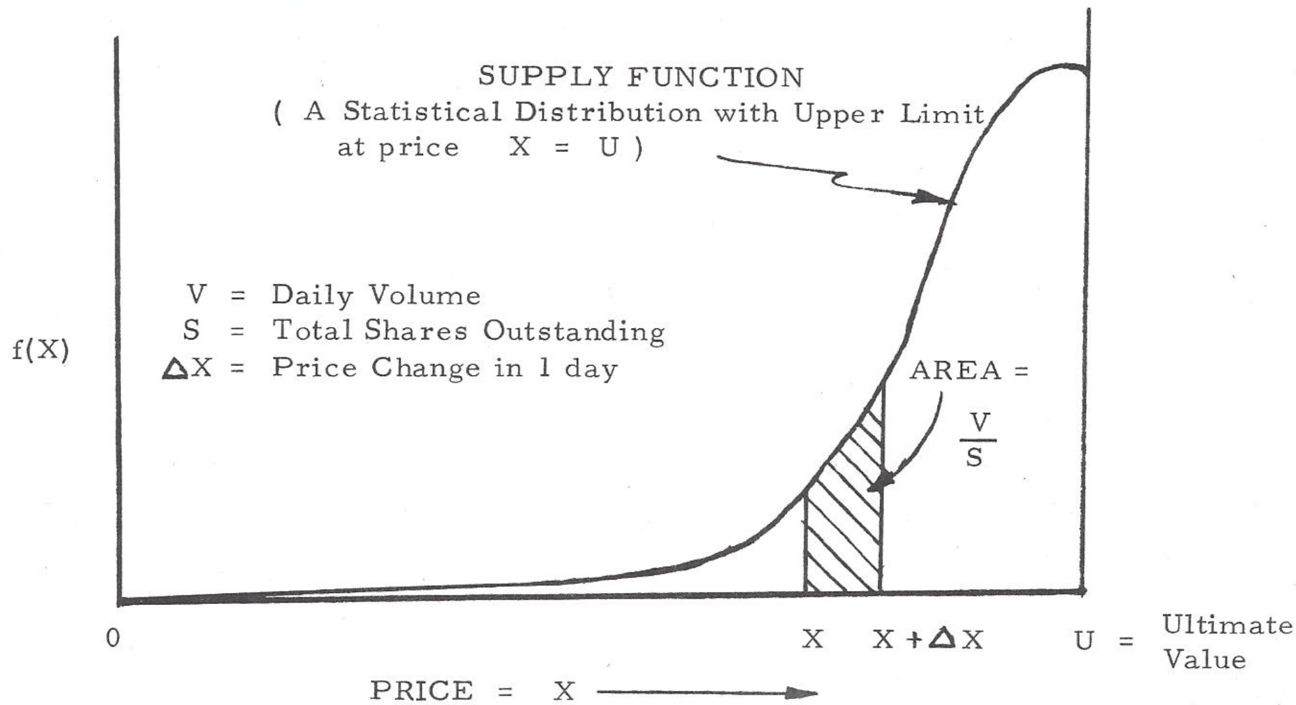
As consequences of this last relation, we can say that

(A) Small volume during high earnings indicates an undervalued stock.

(B) If earnings increase while volume remains fixed, the stock is undervalued.

(C) If volume increases while earnings remain fixed, the stock is overvalued.

THE CONCEPT OF STATISTICAL POTENTIAL



$$\frac{V}{S} = f_{ave.}(X)\Delta X \qquad (\Delta X \text{ is monotonic})$$

$$\frac{V}{S \cdot \Delta X} = f_{ave.}(X)$$

DEFINITION: STATISTICAL POTENTIAL AT

$$X = \ln\left[\frac{1}{\int_0^X f(X)\,dX}\right] = \ln\frac{1}{F(X)} = \psi(X)$$

POTENTIAL FUNCTION $= \psi(X) = -\ln F(X)$

From this : $F(X) = e^{-\psi(X)}$

(See PRINCIPLE #18 for further discussion of the potential function and its related supply curve.)

PRINCIPLES DERIVABLE FROM SUPPLY CURVE THEORY

PRINCIPLE # 1 : In a supply distribution the pole of potential is at the ultimate value.

PRINCIPLE # 2 : Daily volume divided by total shares outstanding represents the total area generated under the supply curve by all price movements between two consecutive closing prices.

PRINCIPLE # 3 : The farther the market price X is from the ultimate value U, the higher the statistical potential of the price, i.e., it is bound to snap back some day --- not saying how soon. Thus, timing is important!

PRINCIPLE # 4 : Little price travel on very large volume indicates that a stock has reached its ultimate level and should be sold, since the statistical potential is practically zero. This type of behaviour must be observed several days in succession to be significant.

PRINCIPLE # 5 : Very little volume on a large price travel (+ or - doesn't matter) indicates that a stock has high statistical potential and should be bought for long term investment.

(No statistical guarantees exist for short term traders.) Such purchases promise more immediate returns if made on an upswing (i.e., when price changes are + .)

PRINCIPLE # 6 : Large volume on a moderate monotone price travel in the upward direction shows that the price is definitely on the way up toward the ultimate value U, i.e., the elastic has snapped back toward lower statistical potentials.

PRINCIPLE # 7 : Upward price movements are confirmed by increased volume for fixed price increments, i.e., by decreases in statistical potential.

PRINCIPLE # 8 : Upward price moves are confirmed by decreases in potential. This means that there is negative potential velocity, i.e., $\frac{d\Psi}{dX}$ is negative.

PRINCIPLE # 9 : Downward price moves are confirmed by increases in potential. This means that there is positive potential velocity , i. e. ,

$$\frac{d\psi}{dX} \text{ is positive .}$$

PRINCIPLE #10: A stock is bottoming out or topping out when the potential velocity is zero.

PRINCIPLE #11: Further drastic price increases are not generally to be expected if the statistical potential has dropped down to .69315, since this is the MEDIAN LEVEL in the supply curve. In any case, at the top of a move, the potential velocity becomes zero, i.e.,

$$\frac{d\psi}{dX} \text{ changes from } - \text{ to } + .$$

PRINCIPLE #12: The bottom of a move can be detected by the cessation of further increases in potential, followed by the start of a drop in potential , i. e., the potential velocity changes sign from + to - .

PRINCIPLE #13: Statistical potential can be expressed in terms of ON BALANCE VOLUME by means of the formula

$$\psi = \ln\left(\frac{2\,S}{S + OBV}\right)$$

where

S = Total Shares Outstanding

OBS = ON BALANCE VOLUME

= Sum of Uptick Volumes - Sum of Downtick Volumes

NOTE: According to this formula, the potential ψ is infinite when OBV = - S, and the potential is zero when OBV = + S .

PRINCIPLE #14: The cumulative supply curve is given by the formula

$$F(X) = .5(1 + OBV/S)$$

Thus,

$$F(X) = e^{-\psi(X)} = e^{-\ln\left(\frac{2S}{OBV + S}\right)}$$

$$F(X) = e^{\ln\left(\frac{OBV + S}{2S}\right)} = \frac{OBV + S}{2S}$$

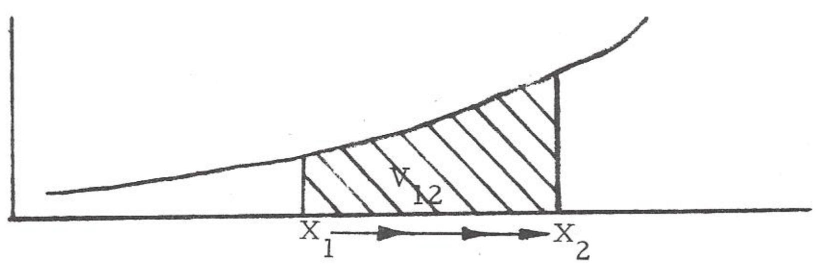
$$F(X) = 1/2 + 1/2\left(\frac{OBV}{S}\right)$$

Therefore, the PDF of the supply curve is

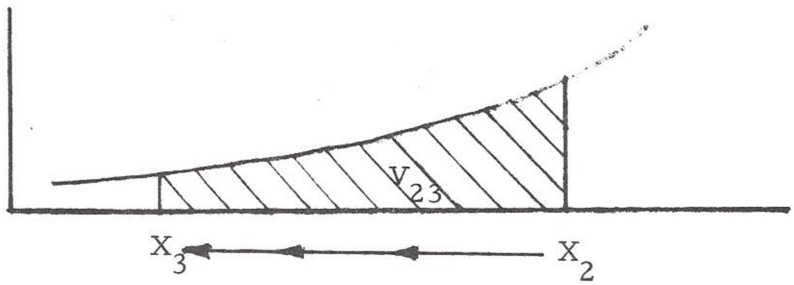
$$f(X) = \frac{1}{2S} \cdot \frac{d(OBV)}{dX}$$

PRINCIPLE #15: Volume under a supply curve is generated by total travel of price, regardless of direction.

For example, if the price goes from X_1 to X_2, the volume generated is V_{12}:



Thus, if the price moves back to X_3, some more volume V_{23} is generated:



The TOTAL VOLUME generated by the TOTAL TRAVEL OF PRICE from X_1 to X_2 to X_3 is then $V_{12} + V_{23}$.

In this case, $\text{TOTAL PRICE TRAVEL} = |X_2 - X_1| + |X_3 - X_2|$

From PRINCIPLE #15 it is possible to have a large volume and no net increase in price, simply by making $X_3 = X_1$. (i.e., Final Price = Initial Price)

PRINCIPLE #16: The important thing to watch each day is the quotient

$$\Lambda = \frac{\text{VOLUME}}{\text{TOTAL PRICE TRAVEL}}$$

This tells us where the market value is along the abscissa of the supply curve.

A high Λ means low potential.

A low Λ means high potential.

PRINCIPLE #17: $$f_{ave.}(X) = \frac{1}{S}\left(\frac{\text{VOLUME}}{\text{TOTAL PRICE TRAVEL}}\right)$$

for any period under consideration.

REMEMBER: Total price travel is not net price change.

For example, if $V = V_1 + V_2 + V_3$ then TOTAL PRICE TRAVEL is $\Delta = |\Delta_1| + |\Delta_2| + |\Delta_3|$, where Δ_1 generates V_1, Δ_2 generates V_2, and Δ_3 generates V_3.

Then in this case, $\text{Average } V_i = \frac{1}{3}(V_1 + V_2 + V_3)$

$$\text{Average}|\Delta_i| = \frac{1}{3}(|\Delta_1| + |\Delta_2| + |\Delta_3|)$$

$$\frac{\text{Average } V_i}{\text{Average}|\Delta_i|} = \frac{V_1 + V_2 + V_3}{|\Delta_1| + |\Delta_2| + |\Delta_3|} = \frac{V}{\Delta} = \frac{\text{VOLUME}}{\text{TOTAL PRICE TRAVEL}}$$

NOTE: This is not the same as $\Lambda_{ave.}$, but it is close to it.

PRINCIPLE #18: The POTENTIAL FUNCTION $\psi(X)$ can be defined by the formula

$$\psi(X) = \left(\frac{U}{X}\right)^{\gamma} - 1 \qquad (\gamma > 0)$$

where U is the ULTIMATE VALUE.

From this definition of the POTENTIAL FUNCTION we obtain the CDF of the supply curve as

$$F(X) = e^{-\psi(X)} = e^{1 - \left(\frac{U}{X}\right)^{\gamma}}$$

Solving F(X) for X :

$$\left\{ \begin{array}{lcl} F(0) & = & 0 \\ F(U) & = & 1 \end{array} \right\}$$

$$X = \frac{U}{(1 - \ln F)^{1/\gamma}}$$

The PDF of the supply curve is

$$f(X) = \frac{d}{dX} F(X) = \frac{\gamma}{X}\left(\frac{U}{X}\right)^{\gamma} e^{1 - \left(\frac{U}{X}\right)^{\gamma}}$$

$$f(X) = \frac{\gamma}{X}\left[1 + \psi(X)\right] e^{-\psi(X)} = \frac{\gamma}{X}\left[1 + \psi(X)\right] F(X)$$

$$\left\{ \begin{array}{lcl} f(0) & = & 0 \\ f(U) & = & \gamma / U \end{array} \right\}$$

The MODE of this frequency curve is at $X_{mode} = \dfrac{U}{\left(1 + \frac{1}{\gamma}\right)^{1/\gamma}}$

(For $\gamma = \infty$: $X_{mode} = U$)

Furthermore, $\psi(X_{mode}) = \dfrac{1}{\gamma}$

$$F(X_{mode}) = e^{-\frac{1}{\gamma}}$$

$$f(X_{mode}) = \frac{\gamma}{U}\left(1 + \frac{1}{\gamma}\right)^{\left(1 + \frac{1}{\gamma}\right)} e^{-\frac{1}{\gamma}}$$

= MODAL ORDINATE

Also, $f(U) = \frac{\gamma}{U}$ = FINAL ORDINATE (of frequency curve) at ULTIMATE VALUE.

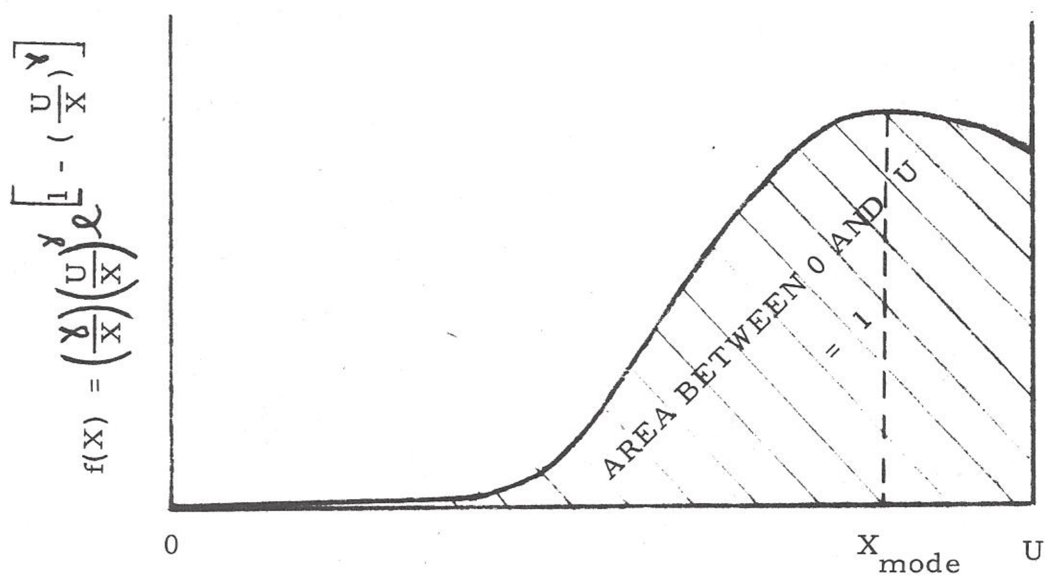
Thus, MODAL ORDINATE = (FINAL ORDINATE) x $\left(1+\frac{1}{\gamma}\right)^{\left(1+\frac{1}{\gamma}\right)} e^{-\frac{1}{\gamma}}$

TABLE OF RATIO $\left(\frac{\text{MODAL ORDINATE}}{\text{FINAL ORDINATE}}\right)$ for various SHAPE PARAMETERS

SHAPE PARAMETER γ	$\frac{\text{MODAL ORDINATE}}{\text{FINAL ORDINATE}}$
1	1.4715
2	1.1143
3	1.0515
10	1.0048
100	1.00005

The SUPPLY CURVE (PDF) defined by the POTENTIAL FUNCTION

$\psi(X) = \left(\frac{U}{X}\right)^{\gamma} - 1$ looks as follows :



The MAXIMUM SUPPLY is at X_{mode}, which is not very much less than the ULTIMATE VALUE, especially if the SHAPE PARAMETER γ is 4 or more. In fact, for $\gamma = 4$, the mode is about 95% of the ULTIMATE VALUE.

For very large values of γ, the mode coincides with the ultimate value. For $\gamma = 1$, the mode is HALF of the ultimate value.

The general relation is $$\frac{X_{mode}}{U} = (1 + \gamma)^{-\frac{1}{\gamma}}$$

NOTE: A constant U and γ indicate a stable supply curve.

A shifting U and a constant γ indicate a stretching or contracting supply curve of fixed shape.

U is the SCALE PARAMETER.

γ is the SHAPE PARAMETER.

A change in γ indicates a change in the ratio $\left(\frac{X_{mode}}{U}\right)$, i.e., a change in the shape of the supply pdf.

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BASIC PROBLEM IN APPRAISING A GIVEN STOCK

For purpose of timing purchases or sales of a given security, it is important to know two things :

(1) The QUANTILE LEVEL of the present market value X on the SUPPLY CURVE.

(2) The DIRECTION of movement of the quantile level.

Obviously, a LOW QUANTILE LEVEL for a security's market value indicates a HIGH POTENTIAL, since the price is far removed to the left of the ULTIMATE VALUE U.

If we are going to buy a stock at a price X, we want the QUANTILE LEVEL of X to be low, and we also want to be assured that the quantile level is increasing, i. e., that the price is definitely going up, and that the situation is technically bullish.

A low quantile level which is on the increase holds promise of appreciation and growth.

The quantile level for a market price X is related to :

(1) The supply curve parameters (U and γ).

(2) The supply curve ordinate at X, i. e., f(X).

Knowing the supply curve parameters U and γ, it is possible to calculate the quantile level of X, which is simply

$$F(X) = e^{1 - \left(\frac{U}{X}\right)^{\gamma}}$$

The difficulty in practice is that U and γ are unknown, and must first be estimated from values of f(X) given by $\frac{1}{S}\left(\frac{\text{VOLUME}}{\text{TOTAL PRICE TRAVEL}}\right)$

(S = Total Shares Oustanding)

A very simple way of estimating the parameters U and γ is to plot actual price data on ULTIMATE VALUE PROBABILITY PAPER. An actual example of this is given on the next page.

ESTIMATING THE SUPPLY CURVE BY PLOTTING PRICES ON ULTIMATE VALUE PROBABILITY PAPER

NUMERICAL EXAMPLE

INPUT DATA :

During 1974 the monthly closing prices of MINNESOTA MINING AND MANUFACTURING (3M) were as follows :

76, 76 1/4, 74 1/4, 71 1/2, 70 5/8, 73 1/2, 65, 55 1/4, 49, 61 5/8, 52 3/4, and 44 5/8 (December low) .

TABLE OF PLOTTING COORDINATES

ORDER STATISTIC NO.	3M PRICE	MEDIAN RANK
1	44.625	5.6%
2	49	13.7%
3	52.75	21.8%
4	55.25	29.8%
5	61.625	37.9%
6	65	46.0%
7	70.625	54.0%
8	71.5	62.1%
9	73.5	70.2%
10	74.25	78.2%
11	76	86.3%
12	76.25	94.4%

Plotting these 12 points on ULTIMATE VALUE PROBABILITY PAPER yields FIGURE 1 on the next page.

From FIGURE 1 we see that Shape Parameter γ = 2.15 and
ULTIMATE VALUE U = $83

It is also apparent from FIGURE 1 that at $50 or less 3M is a good investment once it starts rising, and that it can safely be retained until it reaches $75, which is the 80th percentile in the supply curve. If later data show that the ultimate value exceeds $83, then the stock can be safely held even beyond $75.

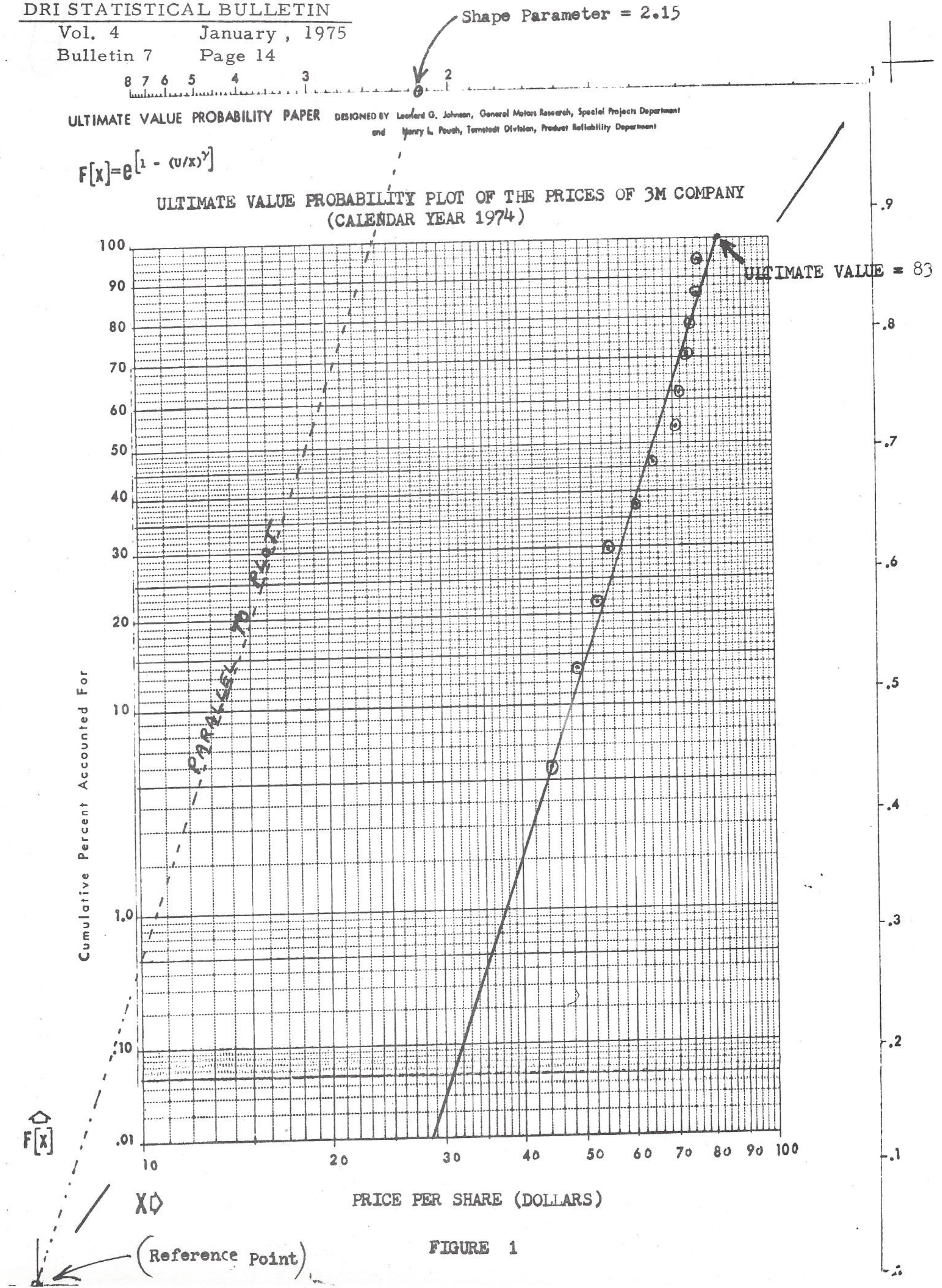
Shape Parameter = 2.15
8 7 6 5 4 3 2 1
ULTIMATE VALUE PROBABILITY PAPER
DESIGNED BY Leonard G. Johnson, General Motors Research, Special Projects Department and Henry L. Pouch, Ternstedt Division, Product Reliability Department
F[X]=e[1 - (U/X)γ]
ULTIMATE VALUE PROBABILITY PLOT OF THE PRICES OF 3M COMPANY
(CALENDAR YEAR 1974)
ULTIMATE VALUE = 83
PARALLEL TO PLOT
Cumulative Percent Accounted For
100
90
80
70
60
50
40
30
20
10
1.0
.10
.01
.9
.8
.7
.6
.5
.4
.3
.2
.1
F[X]
10
20
30
40
50
60
70
80
90
100
XΦ
PRICE PER SHARE (DOLLARS)
(Reference Point)

FIGURE 1