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Reliability & Variation Research

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FIELD SURVEILLANCE RELIABILITY FORMULAS

DEFINITIONS AND NOTATION

K = Number of months after start of Sales Year

S_K = Cumulative Sales through $K^{\underline{th}}$ month after start of Sales Year

S_{12} = Projected Annual Sales

S_1 = Sales Per Month

F_K = Cumulative Failure Cases through $K^{\underline{th}}$ Month after start of Sales Year

R (Annual) = Annual Reliability per Vehicle (for the item in question)

θ = Characteristic Life (At which 63.2% of items have failed)

W = Cumulative Weeks from start of Sales Year = $4.33\,K$

S_W = Cumulative Sales through $W^{\underline{th}}$ week after start of Sales Year

F_W = Cumulative Failure Cases through $W^{\underline{th}}$ week after start of Sales Year

b = Weibull Slope of the failure distribution of the item

For $0 < K \leq 12$ Months

ASSUMPTIONS :

(1) Item has an exponential distribution of times to failures (b=1)*

(2) Average owner drives 1000 miles per month

(3) Minimum Life = 0 (Since failure is possible at zero miles)

$$R(\text{Annual}) = e^{-\frac{24F_K}{KS_K}}$$

$$\theta(\text{in Miles}) = \frac{500KS_K}{F_K}$$

$$\theta(\text{in Months}) = \frac{KS_K}{2F_K}$$

ASSUMING UNIFORM MONTHLY SALES : $\frac{S_{12}}{S_K} = \frac{12}{K}$

$$\therefore \quad R(\text{Annual}) = e^{-\frac{288F_K}{K^2S_{12}}}$$

$$\theta(\text{in Miles}) = \frac{500K^2S_{12}}{12F_K};$$

$$\theta(\text{in Months}) = \frac{K^2S_{12}}{24F_K}$$

* Generally speaking , this is true of any complex assembly made up of many parts .

$K \geqslant 12$ Months

For b = Weibull Slope = 1 :

$$R(\text{Annual}) = e^{-\left(\frac{12}{S_{12}}\right)\left(\frac{F_K}{K-6}\right)}$$

$$\theta\,(\text{in Miles}) = \frac{1000\, S_{12}(K-6)}{F_K}$$

$$\theta\,(\text{in Months}) = \frac{S_{12}(K-6)}{F_K}$$

NOTE : S_{12} = Total Sales for the Model Year

(After 12 months S_{12} is actually known , but before 12 months it must be estimated .)

APPENDIX

MORE GENERAL CASE : (FOR WEIBULL SLOPE = b (any positive number)

ASSUMPTIONS : (1) Average owner drives 1000 miles per month

(2) Minimum Life of the Item = 0

(3) Uniform monthly sales $= \left(\frac{S_{12}}{12}\right)$ units/month

CASE I : ($0 \leq K \leq 12$ Months)

$$R\text{ (Annual)} = e^{-\left[\frac{12 \cdot F_K \cdot (24)^b}{1 + 3^b + 5^b + \ldots + (2K-1)^b}\right]\left(\frac{1}{S_{12}}\right)}$$

$$\theta\text{ (in Miles)} = \frac{500}{12^{1/b}} \cdot \left(\frac{S_{12}}{F_K}\right)^{1/b} \cdot \left[1 + 3^b + 5^b + \ldots + (2K-1)^b\right]^{1/b}$$

CASE II : ($K \geq 12$ Months)

$$R\text{(Annual)} = e^{-\left[\frac{12^{b+1} \cdot F_K}{(K-.5)^b + (K-1.5)^b + (K-2.5)^b + \ldots + (K-11.5)^b}\right]\left(\frac{1}{S_{12}}\right)}$$

$$\theta\text{ (in Miles)} = \frac{1000}{12^{1/b}} \left(\frac{S_{12}}{F_K}\right)^{1/b} \cdot \left[(K-.5)^b + (K-1.5)^b + (K-2.5)^b + \ldots + (K-11.5)^b\right]^{1/b}$$

MODIFICATIONS WHEN MILEAGES OF FAILED ITEMS ARE KNOWN

$$R(\text{Annual}) = e^{-\left[\dfrac{12000\,F_K}{500\,K\,(S_K - F_K) + \sum_{i=1}^{F_K} X_i}\right]} \qquad \left\{\begin{array}{c}(0 < K \leq 12 \text{ Mo.})\\ (b = 1)\end{array}\right\}$$

$$\theta\,(\text{In Miles}) = \frac{\sum_{i=1}^{F_K} X_i + 500\,K\,(S_K - F_K)}{F_K}$$

NOTE : The mileages of the failures are $X_1, X_2, \ldots, X_{F_K}$.

$$R(\text{Annual}) = e^{-\left[\dfrac{12000\,F_K}{1000\,(S_{12} - F_K)(K - 6) + \sum_{i=1}^{F_K} X_i}\right]} \qquad \left\{\begin{array}{c}(K \geq 12 \text{ Months})\\ (b = 1)\end{array}\right\}$$

$$\theta\,(\text{In Miles}) = \frac{\sum_{i=1}^{F_K} X_i + 1000\,(S_{12} - F_K)(K - 6)}{F_K}$$

FOR GENERAL WEIBULL SLOPE b, AND MILEAGES OF THE FAILURES KNOWN TO BE $(X_1, X_2, \ldots, X_{F_K})$

CASE I: $0 < K \leq 12$ Months

$$\theta\text{(In Miles)} = \left\{ \frac{\sum_{i=1}^{F_K} X_i^b + 500^b \left(\frac{S_K - F_K}{K} \right) \left[1 + 3^b + 5^b + \ldots + (2K-1)^b \right]}{F_K} \right\}^{1/b}$$

$$R\text{ (Annual)} = e^{-\dfrac{12000^b F_K}{\sum_{i=1}^{F_K} X_i^b + 500^b \left(\frac{S_K - F_K}{K} \right) \left[1 + 3^b + 5^b + \ldots + (2K-1)^b \right]}}$$

CASE II: $K \geq 12$ Months

$$\theta\text{(In Miles)} = \left\{ \frac{\sum_{i=1}^{F_K} X_i^b + 1000^b \left(\frac{S_{12} - F_K}{12} \right) \left[(K-.5)^b + (K-1.5)^b + \ldots + (K-11.5)^b \right]}{F_K} \right\}^{1/b}$$

$$R\text{(Annual)} = e^{-\dfrac{12000^b F_K}{\sum_{i=1}^{F_K} X_i^b + 1000^b \left(\frac{S_{12} - F_K}{12} \right) \left[(K-.5)^b + (K-1.5)^b + \ldots + (K-11.5)^b \right]}}$$