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A RELIABILITY INDEX BASED ON PART SALES

PROBLEM: To develop an index for comparing the sales of a part in successive periods, such that the index can be used as a warning of trouble whenever it occurs.

DESIDERATA FOR THE INDEX

(1) The index should work in spite of seasonal fluctuations.

(2) The index should have a logical statistical basis.

(3) The theoretical distribution function for the index should be known, in order that its significance levels may be evaluated.

HOW TO REALIZE THE DESIDERATA

Desideratum (1) can be realized by using a MOVING AVERAGE over an integral number of years, as long as seasonal effects are fixed year after year.

Desideratum (2) can be realized by making the index an ARITHMETIC MEAN, expressing the MEAN TIME BETWEEN REPLACEMENTS for the part being studied in the vehicles in which it is used.

Desideratum (3) can be realized by using the property of ASYMPTOTIC NORMALITY for arithmetic averages based on large sample sizes.

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A SPECIAL INDEX AND ITS SIGNIFICANCE LEVELS

In order to realize desiderata (1) , (2) , and (3) , let us use a 3-year moving average as follows :

Suppose the index is to be evaluated at the end of MARCH 1973 . We go back to the end of MARCH 1970 , as in FIGURE 1 :

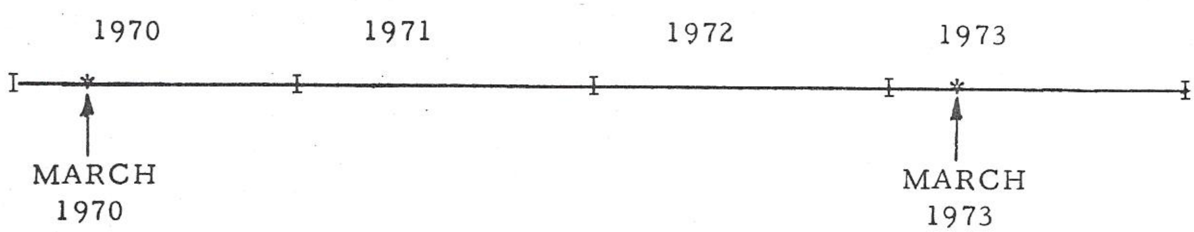

FIGURE 1

We have :

In 1970 : 9 months of vehicle production

In 1971 : 12 months of vehicle production

In 1972 : 12 months of vehicle production

In 1973 : 3 months of vehicle production

Furthermore ,

Vehicles produced in 1970 have an average age (on the road) of 31. 5 months as of the end of MARCH 1973 .

Vehicles produced in 1971 have an average age (on the road) of 21 months as of the end of MARCH 1973 .

Vehicles produced in 1972 have an average age (on the road) of 9 months as of the end of MARCH 1973 .

Vehicles produced in 1973 have an average age (on the road) of 1. 5 months as of the end of MARCH 1973 .

Therefore, during the 3 years from the end of MARCH 1970 to the end of MARCH 1973, the total VEHICLE MONTHS OF USE is proportional to

$$(9 \times 31.5) + (12 \times 21) + (12 \times 9) + (3 \times 1.5) = 648 ,$$

assuming a uniform rate of monthly and annual production of the vehicle population involved.

Now, suppose the total volume of part sales during this same 3 year period (from 3-31-70 to 3-31-73) is V_3 (for a particular part used in the vehicles).

We then take as an INDEX of MEAN TIME BETWEEN REPLACEMENTS, the number

$$I = \frac{648}{V_3} ,$$

which is proportional to the MEAN MONTHS BETWEEN REPLACEMENTS, assuming a uniform monthly production rate.

The STANDARD ERROR of I can be determined by using the fact that each replacement is a RANDOM EVENT in an exponential distribution of replacement times, with a sample size equal to the part sales volume V_3.

Thus,

$$\sigma_I = \frac{I}{\sqrt{V_3}} = \frac{648}{V_3 \sqrt{V_3}} .$$

Now, suppose we obtain a similar index at the end of JUNE 1973. We would use the 3 year period from 6-30-70 to 6-30-73. Suppose that in the 3 year period from 6-30-70 to 6-30-73 the part sales volume for the same part is V'_3.

The index then would be

$$I' = \frac{648}{V'_3} .$$

Furthermore , the STANDARD ERROR of I' is

$$\sigma_{I'} = \frac{I'}{\sqrt{V'_3}} = \frac{648}{V'_3 \sqrt{V'_3}} .$$

Now we can set up a SIGNIFICANCE MEASURE for the difference (I' - I) as follows :

$$\text{Define } Z_c = \frac{(I' - I)}{\sigma_{(I' - I)}} ,$$

where $\sigma_{(I' - I)}$ = STANDARD ERROR of the difference $(I' - I) = \sqrt{\sigma_I^2 + \sigma_{I'}^2}$

Putting $I = \frac{648}{V_3}$ and $I' = \frac{648}{V'_3}$,

and putting $\sigma_I = \frac{648}{V_3 \sqrt{V_3}}$ and $\sigma_{I'} = \frac{648}{V'_3 \sqrt{V'_3}}$,

the Z- SCORE MEASURE Z_c becomes

$$Z_c = \frac{\frac{1}{V'_3} - \frac{1}{V_3}}{\sqrt{\frac{1}{V'^3_3} + \frac{1}{V^3_3}}}$$

This Z-SCORE MEASURE will serve as a warning of a significant jump in part sales volume (from V_3 to V'_3) whenever Z_c is a sufficiently LARGE NEGATIVE NUMBER .

For example ,

when $Z_c = -1.28$, the confidence c = 90 % .

when $Z_c = -1.65$, the confidence c = 95 % .

when $Z_c = -2.33$, the confidence c = 99 % .

Conversely , LARGE POSITIVE values of Z_c of these same magnitudes would indicate at confidence levels of 90 % , 95 % , and 99 % , respectively , that there has been a significant REDUCTION in part sales , so that what may have been serious trouble when the part's volume was V_3 , no longer is serious trouble when the same part's volume is a sufficiently low number V'_3 , which yields LARGE POSITIVE values of the Z-SCORE INDEX Z_c .

A NUMERICAL EXAMPLE

PROBLEM : Suppose that at the ends of two successive quarters , the 3-year part sales volumes were as follows :

$$\left.\begin{aligned} V_3 &= 20,000 \text{ parts sold} \\ V'_3 &= 20,300 \text{ parts sold} \end{aligned}\right\}$$

How confident can we be that this is a significant increase , which indicates trouble ?

SOLUTION

Evaluate

$$Z_c = \frac{\dfrac{1}{V'_3} - \dfrac{1}{V_3}}{\sqrt{\dfrac{1}{V'^3_3} + \dfrac{1}{V^3_3}}} = \frac{\dfrac{1}{20300} - \dfrac{1}{20000}}{\sqrt{\dfrac{1}{(20300)^3} + \dfrac{1}{(20000)^3}}}$$

$$= -1.49 .$$

In a NORMAL DISTRIBUTION , a Z-SCORE of magnitude 1.49 represents a confidence level of 93.2 % (Answer) .

For 99 % CONFIDENCE , the part sales volume V'_3 would have to be $V'_3 = 20,470$. (With $V_3 = 20,000$ as before) .

Then ,

$$Z_c = \frac{\frac{1}{20470} - \frac{1}{20000}}{\sqrt{\frac{1}{(20470)^3} + \frac{1}{(20000)^3}}} = -2.33 ,$$

which is the Z-SCORE for 99 % confidence .

ANOTHER NUMERICAL EXAMPLE

PROBLEM : Suppose that at the ends of two successive quarters , the 3-year part sales volumes for a particular part were as follows :

$$\left. \begin{aligned} V_3 &= 5000 \text{ parts sold} \\ V'_3 &= 5200 \text{ parts sold} \end{aligned} \right\}$$

How confident can we be that this is a significant increase , which indicates trouble ?

SOLUTION

The Z-SCORE index is

$$Z_c = \frac{\frac{1}{5200} - \frac{1}{5000}}{\sqrt{\frac{1}{(5200)^3} + \frac{1}{(5000)^3}}} = -1.98 .$$

In a NORMAL DISTRIBUTION , 1.98 represents the Z-SCORE for the level c = 97.6 % confidence (Answer)

For 99 % confidence, the part sales volume V'_3 would have to be $V'_3 = 5236$, with $V_3 = 5000$ (as before). Then,

$$Z_c = \frac{\frac{1}{5236} - \frac{1}{5000}}{\sqrt{\frac{1}{(5236)^3} + \frac{1}{(5000)^3}}} = -2.33,$$

which is the Z-SCORE for c = 99 % confidence.

WHAT PERCENT INCREASE REPRESENTS 99 % CONFIDENCE ?

It can be seen from the preceding examples, that whenever the increase in part sales volume in two periods of equal length exceeds $3\,{}^1/_3$ times the SQUARE ROOT of the PREVIOUS PERIOD'S VOLUME, the confidence begins to exceed 99 %.

Thus, if $\Delta V > 3\,{}^1/_3 * \sqrt{V_3}$, there is at least 99 % CONFIDENCE OF TROUBLE. ($\Delta V = V'_3 - V_3$)

This condition, therefore, can be written as follows :

$$V'_3 - V_3 > 3\,{}^1/_3 * \sqrt{V_3}$$

or, $$\boxed{V'_3 > V_3 + 3\,{}^1/_3 * \sqrt{V_3}}$$ (For at least 99% confidence of trouble)

According to this, for at least 99 % confidence of trouble, part sales volume over equal periods must show a percent increase of at least ($333/\sqrt{V_3}$) %.

(V_3 = Previous period's part sales volume.)

EXAMPLE OF PERCENT INCREASE

If the PREVIOUS PERIOD'S VOLUME WAS V_3 = 5000 parts sold , the new period's volume must show an increase of

$$\left(\frac{333}{\sqrt{5000}}\right)\% = 4.71\ \% \text{ over } 5000 ,$$

i.e. , 236 over 5000 (as was shown earlier , for 99 % confidence) .

RANKING PARTS FOR CRITICALITY

Parts can be ranked for criticality by listing the magnitudes of NEGATIVE Z-SCORES numerically from the LARGEST NEGATIVE Z-SCORE to the SMALLEST .

A still simpler method (and mathematically equivalent) of ranking parts for criticality is to calculate the CHANGE IN THE SQUARE ROOT OF THE PART SALES VOLUMES over two successive periods of equal length (say 3-years) and a fixed lag (say 3-months) , and calling this change in square root the CRITICALITY INDEX . (See Theorem below.)

GENERAL THEOREM FOR RANDOM WALKS

THEOREM : At a fixed confidence level , the square roots of random walk variables change by a constant amount . (This is a well known rule which is used in the analysis and prediction of expected changes in stock market prices .)

EXAMPLE

When V_3 was 20,000 , V'_3 was 20,470 for 99 % confidence . This is a change of $\sqrt{20470} - \sqrt{20000} = 1.65$ in the square roots .

Similarly , when V_3 was 5000 , V'_3 was 5236 for 99 % confidence . Again , this is a change of $\sqrt{5236} - \sqrt{5000} = 1.65$ in the square roots .

Thus, FOR 99 % CONFIDENCE OF TROUBLE, THE SQUARE ROOT OF THE PART SALES VOLUME MUST INCREASE BY THE CONSTANT AMOUNT OF 1.65. (A BEAUTIFUL ILLUSTRATION OF THE SO-CALLED "OCCAM'S RAZOR" PRINCIPLE IN MATHEMATICAL LOGIC.)

The changes needed in the square roots of part sales volumes for different confidence levels are listed in the table below.

CHANGE IN SQUARE ROOT OF PART SALES VOLUME	CONFIDENCE LEVEL
0	50 %
.18	60 %
.37	70 %
.60	80 %
.73	85 %
.91	90 %
1.16	95 %
1.65	99 %
2.19	99.9%
2.33	99.95%
2.63	99.99%

(See chart on page 14.)

RELIABILITY ESTIMATION FROM PART SALES VOLUME

PROBLEM : Suppose we are looking at a 3-year period (say , from 3-31-70 to 3-31-73) , and suppose that the total sales volume of a particular part is V_3 for this 3-year period . What is the ANNUAL RELIABILITY of the part in the vehicles where it is being used ?

SOLUTION

We make some assumptions :

(1) The oldest age of a vehicle using the part is Y years , i. e. , the part has been in use for the last Y years .

(2) The annual sales total for vehicles using the part has not changed over the last Y years .

We then have two distinct portions of sold vehicles to consider :

PORTION I : Those vehicles sold in the 3 years for which the part sales volume is V_3 .

PORTION II : Those vehicles sold in the (Y - 3) years preceding the 3 years of PORTION I .

In PORTION I , the total vehicles sold is 36 months' worth .
In PORTION II , the total vehicles sold is $12(Y - 3) = (12Y - 36)$ months' worth .

For a vehicle sold in PORTION I , the AVERAGE MONTHS ON THE ROAD is 18 months.
For each vehicle sold in PORTION II , the number of months on the road is 36 months . (Remember , we are only considering the period from A to B .)

Thus , we have a situation depicted in FIGURE 2 on the following page :

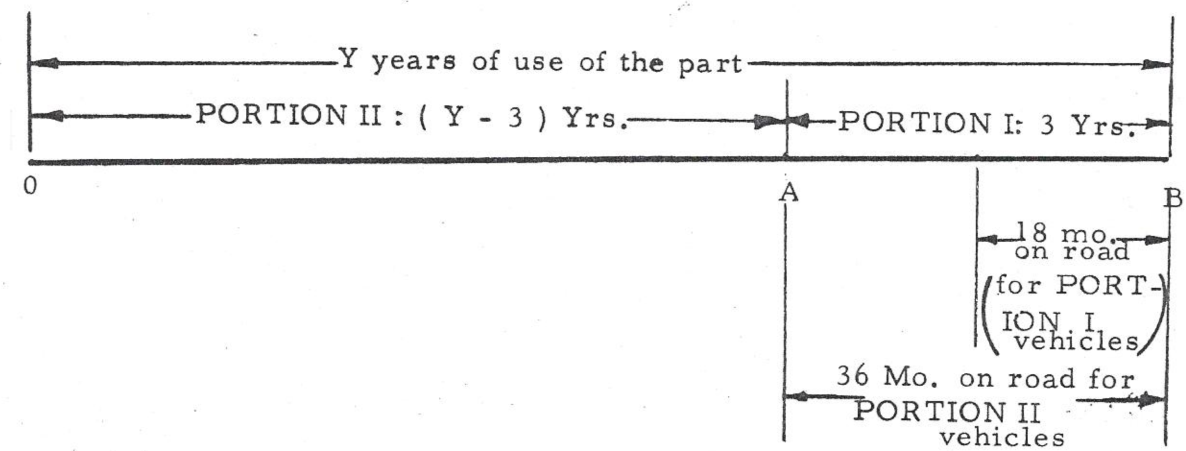

FIGURE 2

From FIGURE 2, we see that the total vehicle months on the road A to B is

$$T = (\text{Monthly Production})\left[36 \times 18 + (12Y - 36) \times 36\right] \quad *$$
$$= 36(12Y - 18)\ (\text{Monthly Production})$$
$$= 3(12Y - 18)\ (\text{Annual Production})$$
$$= 18(2Y - 3)\ (\text{Annual Production})$$

The MEAN MONTHS BETWEEN FAILURES is then (assuming V_3 parts sold between A and B) is

$$\text{MMBF} = \frac{18(2Y - 3)\ (\text{Annual Production})}{V_3}$$

The MONTHLY RELIABILITY (assuming an exponential distribution of times to failure) is then

$$R(1\ \text{Month}) = e^{-\frac{1}{\text{MMBF}}}$$

$$= e^{-\frac{V_3}{18(2Y - 3)\ (\text{Annual Production})}}$$

The ANNUAL RELIABILITY is

$$R(12\ \text{Months}) = e^{-\frac{12}{MMBF}}$$

$$= e^{-\frac{V_3}{(3Y - 4.5)\ (\text{Annual Production})}}$$

NUMERICAL EXAMPLE OF RELIABILITY ESTIMATION

Suppose that the ANNUAL PRODUCTION of vehicles using a certain part is 500, 000 vehicles/yr. , and , suppose , the part has been in use for the last 10 years .

In the 3 years from 3-31-70 to 3-31-73 the number of such spare parts sold was 50, 000 . What is the ANNUAL RELIABILITY ?

SOLUTION

$$R(12\ \text{Months}) = e^{-\frac{V_3}{25.5(\text{Annual Production})}}$$

In this case : $V_3 = 50,000$; $Y = 10$ years

Annual Production = 500, 000

Hence,

$$R(12\ \text{Months}) = e^{-\frac{50,000}{25.5 \times 500,000}} = e^{-\frac{1}{255}}$$

$$= 1 - \frac{1}{255} + \frac{\left(\frac{1}{255}\right)^2}{2!} - \frac{\left(\frac{1}{255}\right)^3}{3!} + \cdots$$

$$= 1 - .00392157 + .00000769 - \cdots$$

$$= 0.99608612 \quad (\text{Answer})$$

RELIABILITY FORMULA FOR A 2-YEAR BASE PERIOD

The formula for ANNUAL RELIABILITY given on page 12 assumes a 3-year base period . If a 2-year base period is used , the formula becomes (by a similar type of analysis as that performed in FIGURE 2) the following :

$$R(12 \text{ Months}) = e^{-\frac{V_2}{2(Y - 1)(\text{Annual Production})}}$$

(V_2 = Part sales volume over the 2-year base period.)

Furthermore , the estimate of MEAN MONTHS BETWEEN FAILURES is

$$\text{MMBF} = \frac{24(Y - 1)(\text{Annual Production})}{V_2}$$

From these expressions , and those on page 11 and page 12 , it can be seen that if the ANNUAL RELIABILITY has not changed , the relation between V_2 and V_3 is

$$\frac{V_2}{V_3} = \frac{2}{3}\left(\frac{Y - 1}{Y - 1.5}\right)$$

This ratio is 2/3 only if Y is infinite , i. e. , only if the part is used indefinitely .

NOTE :

* This assumes no attrition for the total of Y years from 0 to B . If there is attrition , it would be for PORTION II vehicles , where we would multiply (Y - 3) by an attrition coefficient α.
Then ,

$$T = (\text{Monthly Prod.})\left[36 \times 18 + \alpha(12Y - 36) \times 36\right] ; \quad (0 \leq \alpha \leq 1)$$

CONFIDENCE CHART FOR A RANDOM WALK VARIABLE