

DO QUALITY AUDITS REALLY HELP ?

In order to construct a mathematical model which will evaluate the effect of a quality audit on the fraction defective in the field, let us adopt the following notation:

Let i = Model Year

Let $A(i)$ = Audit Fraction Defective in the Model Year i

Let $F_o(i)$ = Field Fraction Defective before audit in Model Year i

Let $F_A(i)$ = Field Fraction Defective after audit in Model year i

Let E_i = Clean-Up Coefficient, i. e., our ability to eliminate the faults uncovered by the audit in Model Year i
 (This includes the probability that a correction at zero miles)
 (will last through all warranty miles.)

ENTIRE FIGURE = TOTAL FIELD POPULATION FOR MODEL YEAR i

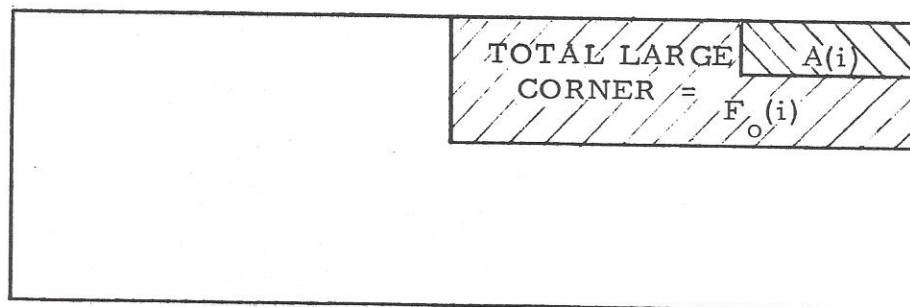


FIGURE 1

IN FIGURE 1 : TOTAL LARGE CORNER = $F_o(i)$
 SMALL CORNER = $A(i)$

$$F_A(i) = F_o(i) - E_i A(i)$$

(Clean-Up Equation)

(1)

$$(0 \leq E_i \leq 1)$$

Assume that the audit includes inspection of the proportion q_i of all potentially defective items in MODEL YEAR i . We can then write the following PROPORTIONALITY EQUATION :

$$A(i) = q_i F_o(i)$$

 $(0 \leq q_i \leq 1)$

(2)

Substituting (2) into (1) :

$$F_A(i) = F_o(i) - E_i q_i F_o(i) = (1 - E_i q_i) F_o(i)$$

(3)

or,

$$F_A(i) = \left(\frac{1 - E_i q_i}{q_i} \right) A(i)$$

or,

$$F_A(i) = \left(\frac{1}{q_i} - E_i \right) A(i)$$

(4)

Examples of AUDIT DATA and WARRANTY DATA for Model Year 1972 and 1973 are shown on page 3. The AUDIT DATA as tabulated represent $A(i)$ for $i = 1972$ and $i = 1973$. The WARRANTY DATA as tabulated represent $F_A(i)$ for $i = 1972$ and $i = 1973$.

<u>'72 AUDIT</u>	<u>'72 WARRANTY</u>	<u>AUDITED ITEM NO.</u>	<u>'73 AUDIT</u>	<u>'73 WARRANTY</u>
0.9%	6.6%	1	2.5%	6.4%
5.4	4.7	2	4.3	5.4
3.1	11.7	3	2.8	11.2
3.5	10.4	4	2.6	13.0
1.0	12.6	5	1.1	14.4
2.1	4.9	6	1.0	9.2
1.6	1.0	7	2.8	0.8
5.8	1.3	8	2.7	1.2
0.9	0.9	9	2.6	3.8
1.7	6.2	10	0.5	6.4
1.3	10.1	11	2.7	13.0
1.4	2.6	12	1.6	2.8
3.4	3.6	13	2.0	4.0
0.6	0.5	14	0.5	0.4
3.0	4.1	15	2.9	6.6
5.3	4.9	16	5.8	7.4
1.6	13.8	17	0.6	14.2
0.2	5.6	18	0.3	3.8
2.3%	1.2%	19	0.2%	1.0%
(N = 19)	(N = 19)		(N = 19)	(N = 19)
AVE. = 2.37%	AVE. = 5.62%		AVE. = 2.08%	AVE. = 6.58%

Thus, during the Model Year 1972 : $F_A('72) = 5.62%$, and $A('72) = 2.37%$

and , during the Model Year 1973 : $F_A('73) = 6.58%$, and $A('73) = 2.08%$

Suppose that all faults uncovered by the audits were corrected and remained corrected throughout warranty , i. e. , $E_{172} = E_{173} = 1$. Obviously, this is a very optimistic assumption . However , let us just suppose such was the case.

Then ,

$$F_{A(i)} = \left(\frac{1}{q_i} - 1 \right) A(i)$$

$$\frac{1}{q_i} - 1 = \frac{F_{A(i)}}{A(i)}$$

$$\frac{1}{q_i} = \frac{F_{A(i)}}{A(i)} + 1$$

$q_i = \frac{1}{\frac{F_{A(i)}}{A(i)} + 1}$	(5)
---	-----

From (5) we obtain :

$$q_{172} = \frac{1}{\frac{5.62}{2.37} + 1} = .29662$$

and , $q_{173} = \frac{1}{\frac{6.58}{2.08} + 1} = .24018$

Using these values of q_{172} and q_{173} in the PROPORTIONALITY EQUATION (2) we obtain :

$$F_o ('72) = \frac{A('72)}{q_{172}} = \frac{2.37}{.29662} = 7.99 \%$$

$$F_o ('73) = \frac{A('73)}{q_{173}} = \frac{2.08}{.24018} = 8.66 \%$$

Thus , auditing with 100 % CLEAN-UP COEFFICIENTS reduces a 1972 EXPECTED FRACTION DEFECTIVE of 7.99 % to 5.62 % , and reduces a 1973 EXPECTED FRACTION DEFECTIVE of 8.66 % to 6.58 % . These represent the GREATEST POSSIBLE benefits we could ever get in these particular cases , since they are based on OPTIMISTIC clean-up assumptions .

Thus , in '72 Models we could at most eliminate 30 % of all original defects , and in '73 Models auditing could at most eliminate 24 % of all original defects .

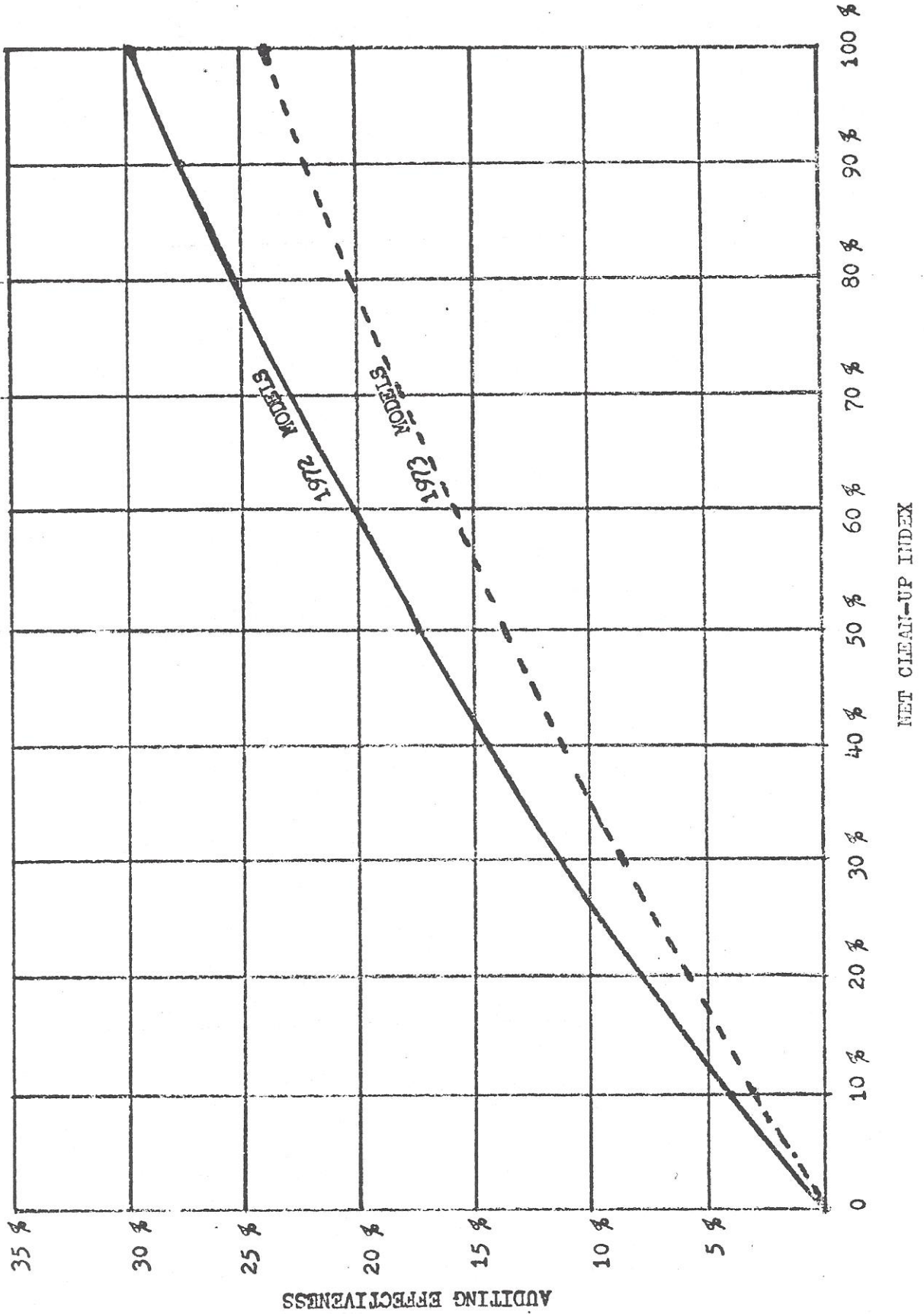
Note that there was an 8 % increase in TOTAL ORIGINAL DEFECTS in 1973 models over 1972 models (i. e. , $8.66 = 1.08 \times 7.99$.) .

CONCLUSION FROM THE EXAMPLE

ASSUMING 100 % CLEAN-UP COEFFICIENTS , THE AUDITING OF CERTAIN ITEMS (#1 thru #19) IN '72 AND '73 MODEL VEHICLES AT ZERO MILES CAN AT MOST REDUCE THE EXPECTED WARRANTY CASES BY 30 % ON '72 MODELS , AND BY 24 % ON '73 MODELS. THE SMALLER BENEFIT (24 %) ON 1973 MODELS CAN BE EXPLAINED BY AN 8% INCREASE OF TOTAL POTENTIAL DEFECTS ON '73 MODELS , WHILE THE AUDITING WAS STILL RESTRICTED TO THE SAME LIST OF ITEMS AS IN THE 1972 MODELS.

FIGURE 2 on page 6 graphically shows the AUDITING EFFECTIVENESS as a function of a NET CLEAN-UP INDEX between 0 and 1. Separate curves have been constructed for the Model Years 1972 and 1973. The mathematical formula for NET CLEAN-UP INDEX and AUDITING EFFECTIVENESS are derived in the APPENDIX .

FIGURE 2 - - - -AUDITING EFFECTIVENESS vs. NET CLEAN-UP INDEX



APPENDIX

MATHEMATICAL OF THE CLEAN-UP INDEX AND AUDITING EFFECTIVENESS

For a particular model year : $\left\{ \begin{array}{l} \text{Let } A_i = \text{Fraction Found Defective In Audit Of Item } \#i \\ \text{Let } E_i = \text{Clean-Up Coefficient for Item } \#i \\ W_i = \text{Warranty Without Audit on Item } \#i \\ W'_i = \text{Warranty With Audit on Item } \#i \end{array} \right.$

On Item # 1 : $W'_1 = W_1 - E_1 A_1$

On Item # 2 : $W'_2 = W_2 - E_2 A_2$

On Item # 3 : $W'_3 = W_3 - E_3 A_3$ (N = Total No. of Items Audited)

.

On Item # N : $W'_N = W_N - E_N A_N$

Summing over all items audited:

$$\sum_{i=1}^N W'_i = \sum_{i=1}^N W_i - \sum_{i=1}^N E_i A_i$$

Dividing through by N :

$$\frac{\sum_{i=1}^N W'_i}{N} = \frac{\sum_{i=1}^N W_i}{N} - \frac{\sum_{i=1}^N E_i A_i}{N}$$

or , $\bar{W}' = \bar{W} - \bar{E A}$ (The bars denote averages.)

NOTE : $\bar{E A}$ is the average of all products of the form $E_i A_i$. This is not in general the same as $\bar{E} \bar{A}$, because the average of a product is not equal to the product of the averages.

However , there always exists a NET CLEAN -UP INDEX \hat{E} , such that $\bar{E} \bar{A} = \hat{E} \bar{A}$. In terms of this NET CLEAN -UP INDEX \hat{E} , we can write

$$\bar{W}' = \bar{W} - \hat{E} \bar{A} \tag{6}$$

From this :

$$\bar{W} - \bar{W}' = \hat{E} \bar{A} \tag{7}$$

AUDITING EFFECTIVENESS is defined by

$$Z = \frac{\bar{W} - \bar{W}'}{\bar{W}}$$

Using (6) and (7) , this can be written as follows :

$$Z = \frac{\hat{E} \bar{A}}{\bar{W}' + \hat{E} \bar{A}}$$

or

$$Z = \frac{1}{1 + \left(\frac{1}{\hat{E}} \right) \left(\frac{\bar{W}'}{\bar{A}} \right)} \tag{8}$$

\hat{E} = NET CLEAN-UP INDEX for the entire system of items audited *

\bar{W}' = AVERAGE FRACTION DEFECTIVE as indicated by WARRANTY AFTER AUDIT

\bar{A} = AVERAGE FRACTION DEFECTIVE as found in the audit

The graphs in FIGURE 2 are based on formula (8) with \hat{E} as abscissa and Z as ordinate .

* CLEAN-UP INDICES really indicate to what extent faults found by audit (at zero miles) are eliminated from the product through all warranty miles .

Thus , auditing alone at zero miles is not enough ----- there must also be RELIABILITY to last through the warranty period.

RELIABILITY can not be inspected into a product.