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THE DUMMY COMPARISON PROBLEM

The use of dummies to establish the levels of protection in automobile occupant protection systems requires that consistent results should be obtained in the same car under repeated tests in which the only change is in the individual dummy used.
In other words, dummy manufacturing should be so well controlled that supposedly identical dummies really do possess identical properties under like conditions of dynamical excitation in the same vehicle. Otherwise, we end up confounding dummy differences with vehicle occupant protection properties. When can we say that two individual dummies are identical enough to permit their usage in evaluating occupant protection levels without having to worry about dummy differences?

We can answer this question by performing separate statistical significance tests of comparison on

(1) Mean head accelerations from identical sled excitations on the two dummies in the same occupant protection system.

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(2) Mean chest accelerations from identical sled excitations on the two dummies in the same occupant protection system.

(3) Femur loads on the two dummies induced by identical sled excitations in the same occupant protection system.

In the discussion which follows we shall outline the mathematics of setting up a numerical measure which indicates what degree of discrepancy exists between two individual dummies, and then we shall proceed to show how this measure can be converted into a CONFIDENCE INDEX that DUMMY B is like DUMMY A.

COMPARING HEAD ACCELERATION RESPONSES ON TWO DUMMIES

FIGURE 1 below shows the MEAN HEAD ACCELERATIONS versus TIME (from 0 milliseconds to 500 milliseconds) on dummies A and B under like conditions

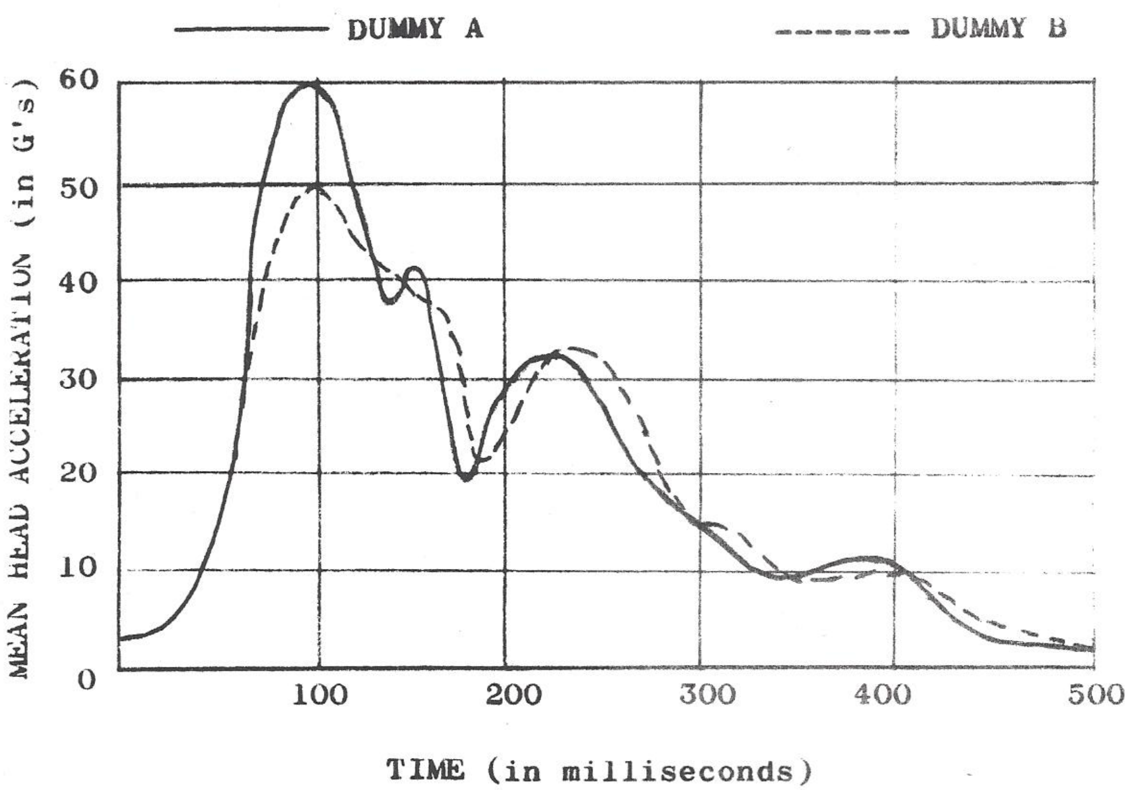

FIGURE 1

QUESTION: How can we measure the extent to which DUMMY B is statistically in agreement with DUMMY A over the entire range from 0 to 500 milliseconds?

ANSWER: Calculate $S = \sum(B - A)^2$ = SUM OF SQUARES OF RESIDUALS over all the time intervals used in constructing the acceleration curves, and then perform a statistical test for the significance of the sum of squared residuals so obtained.

DOES DUMMY B BEHAVE LIKE DUMMY A ?

Taking DUMMY A as the STANDARD , we ask the question: "Does dummy B behave like dummy A?" For illustration, suppose the RAW DATA on A and B are as follows (readings listed for every 10 milliseconds; such readings could be listed for every millisecond, if desired.):

Time (M-Sec.)	Head Accel. (G's) A	Head Accel. (G's) B	Time (M-Sec)	Head Accel. (G's) A	Head Accel. (G's) B
10	2	2	260	10	10
20	3	3	270	11	10
30	4	4	280	11	10
40	5	4	290	11	10
50	6	5	300	12	10
60	10	6	310	13	10
70	25	12	320	10	10
80	38	23	330	9	9
90	59	50	340	8	9
100	60	51	350	7	8
110	55	47	360	6	7
120	38	40	370	5	6
130	30	32	380	5	6
140	25	27	390	5	5
150	20	24	400	4	5
160	21	22	410	4	4
170	24	20	420	4	4
180	27	25	430	3	4
190	29	29	440	3	4
200	32	33	450	3	3
210	30	32	460	3	3
220	25	29	470	2	3
230	20	21	480	2	2
240	14	16	490	2	2
250	10	11	500	2	2

Since there are N = 50 readings for each dummy, we sum up 50 squared differences for $\sum(B - A)^2$. The value of $\sum(B-A)^2$ is then

$S = \sum_{i=1}^{50}(B_i - A_i)^2$, which means that we are to sum up the values obtained by squaring each difference (B - A) (at same no. of M-Sec.).

Thus, we form the sum

$$S = (2-2)^2 + (3-3)^2 + (4-4)^2 + (4-5)^2 + (5-6)^2 + \dots + (2-2)^2 ,$$

where each term in the sum uses the values of A and B at the same no. of milliseconds.

This sum S turns out to be

$$S = 744$$

Furthermore, $\bar{A} = 15.34$ ← Means → $\bar{B} = 14.48$

$\sigma_A = 15.0382$ ← Std. Deviation → $\sigma_B = 13.4624$

(N = 50)

Now, we define a NORMAL Z-SCORE as follows:

$$\text{NORMAL Z-SCORE} = \frac{1}{\sigma_A}\sqrt{\frac{S}{N}}$$

Then ,

$$\text{CONFIDENCE that B differs from A} = 2\left[\eta(Z) - .5\right]$$
$$= 2\eta\left(\frac{1}{\sigma_A}\sqrt{\frac{S}{N}}\right) - 1$$

$\eta(Z)$ = AREA UNDER A NORMAL CURVE FROM $-\infty$ to Z-Score given by Z.

Conversely, we have

$$\text{CONFIDENCE that dummy B is like dummy A} = 1 - 2\eta\left(\frac{1}{\sigma_A}\sqrt{\frac{S}{N}}\right) + 1$$

$$= 2\left[1 - \eta\left(\frac{1}{\sigma_A}\sqrt{\frac{S}{N}}\right)\right]$$

For this particular numerical example, this becomes

$$\text{CONFIDENCE that dummy B is like dummy A} = 2\left[1 - \eta\left(\frac{1}{15.0382}\sqrt{\frac{744}{50}}\right)\right]$$

$$= 2\left[1 - \eta(.2566)\right]$$

$$= 2(1 - .6013) = .7974$$

$$= 79.74\%.$$

Since this is less than 90% confidence, we decide that dummy B is not like dummy A (for head accelerations).

In terms of the Z Score itself we can state the following as the condition for at least 90% confidence that B is like A :

$$\boxed{\frac{1}{\sigma_A}\sqrt{\frac{S}{N}} \leq \frac{1}{8}} \longleftarrow \left\{\text{CONDITION FOR AT LEAST 90\% CONFIDENCE THAT DUMMY B IS LIKE DUMMY A.}\right\}$$

Similar analyses could be made for CHEST ACCELERATIONS and FEMUR LOADS on dummies A and B.

For convenience, we have constructed FIGURE 2, which gives the CONFIDENCE corresponding to a given value of

$$Z = \frac{1}{\sigma_A}\sqrt{\frac{S}{N}}.$$

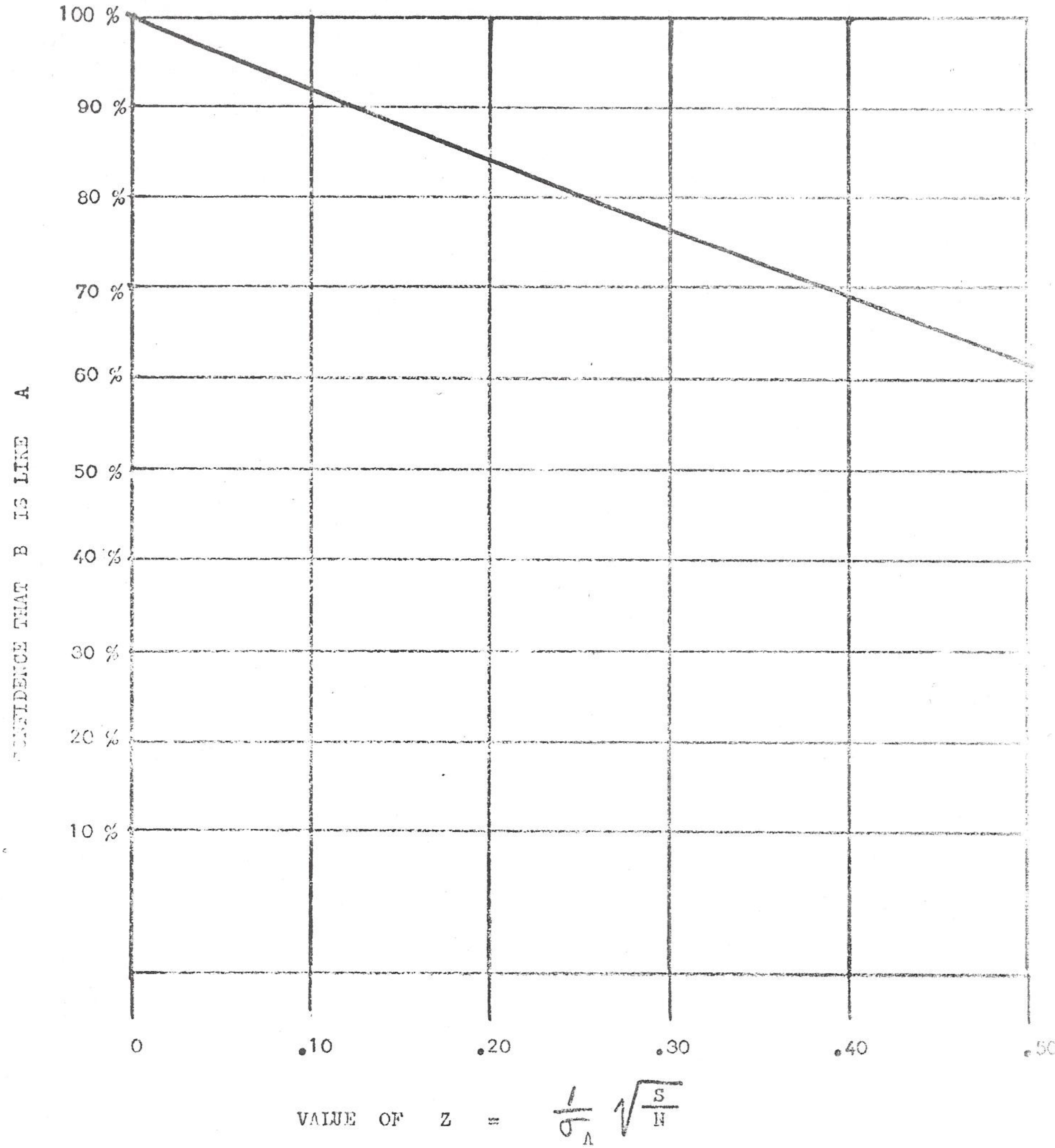
100 %
90 %
80 %
70 %
60 %
50 %
40 %
30 %
20 %
10 %
CONFIDENCE THAT B IS LIKE A
0
.10
.20
.30
.40
.50
VALUE OF Z =

FIGURE 2