Statistical Bulletin
Reliability & Variation Research

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THE SAMPLE SIZE PROBLEM

- WHAT IS THE REQUIREMENT FOR AN "ACCEPTABLE" DESIGN?
- WHAT IS THE COST OF TESTING A SAMPLE OF SIZE N?
- WHAT WOULD BE YOUR POTENTIAL LOSS IF YOU RELEASED AN "UNACCEPTABLE" DESIGN?

ONCE ONE IS ABLE TO ANSWER THESE QUESTIONS RELATIVE TO A PARTICULAR DESIGN THEN THE NECESSARY INFORMATION IS AVAILABLE TO DETERMINE AN OPTIMUM SAMPLE SIZE THAT INCLUDES ECONOMIC CONSIDERATIONS.

THUS, THE PURPOSE OF THIS ISSUE IS TO MAKE YOU AWARE OF AN EXTENDED APPROACH TO THE SAMPLE SIZE PROBLEM. THIS WILL BE ILLUSTRATED BY MEANS OF EXAMPLES.

LET US DISCUSS SOME BASIC IDEAS BEFORE ATTACKING THE SAMPLE SIZE PROBLEM.

ONE OF THE PROBLEMS WE FACE IS TO JUDGE HOW ACCURATELY A FEW TEST VEHICLES (SAMPLE) REPRESENT THE ENTIRE (POPULATION) PRODUCTION RUN.

THE PROBLEM ARISES BECAUSE LIKE DESIGNED VEHICLES WILL HAVE MANY VARIATIONS. SOME OF THE VARIATION IS INTENTIONAL, AS WITH OPTIONAL EQUIPMENT. BUT EVEN WITH IDENTICAL EQUIPMENT, NO TWO THINGS ARE EXACTLY ALIKE.

VARIATIONS TEND TO FALL IN PARTICULAR PATTERNS OR DISTRIBUTIONS. MATHEMATICS OF PROBABILITY SHOW THAT THE GREATER THE NUMBER OF VARIABLES WHICH INFLUENCE THE OUTCOME, THE MORE LIKELY WE WILL SET A PATTERN THAT CLOSELY APPROXIMATES THE NORMAL DISTRIBUTION. FOR THIS TYPE DISTRIBUTION, THE MAJORITY OF THE VALUES FALL CLOSE TO THE AVERAGE VALUE AND VARY IN A SYMMETRICAL MANNER AROUND THE AVERAGE. THE AVERAGE VARIATION AROUND THE CENTRAL VALUE IS CALLED THE STANDARD DEVIATION.

WE ARE NEXT CONCERNED WITH HOW GOOD ARE THE ESTIMATES. THE ACCURACY OF ANY ESTIMATE IS BASED ON A NUMBER OF CONSIDERATIONS. FOR EXAMPLE, IS THE DATA WE HAVE REPRESENTATIVE OF THE POPULATION? IS THE SAMPLE LARGE ENOUGH? STATISTICS ALLOWS US TO CALCULATE A CONFIDENCE LEVEL (PROBABILITY OF BEING RIGHT). TWO INTERESTING OBSERVATIONS FOLLOW:

- FOR THE SAME SAMPLE SIZE, INCREASING THE CONFIDENCE LEVEL WILL EXPAND OUR VARIATION ESTIMATE.

- FOR THE SAME CONFIDENCE LEVEL, INCREASING THE SAMPLE SIZE WILL NARROW OUR VARIATION ESTIMATE.

FINALLY, FOR A GIVEN DESIGN DEFINE THE LOSS TO BE THE SUM OF THE FAILURE AND TEST EXPENSE ASSOCIATED WITH AN ACCEPTABLE AND UN-ACCEPTABLE DESIGN. THEN THE EXPECTED VALUE OF THE LOSS IS DEFINED AS THE RISK ASSOCIATED WITH THAT DESIGN.

NOW LET US EXAMINE IN MORE DETAIL THE FIRST QUESTIONS THAT WERE PRESENTED AT THE START OF THE ISSUE.

- WHAT IS THE REQUIREMENT FOR AN "ACCEPTABLE" DESIGN?

WHEN WE CALL A DESIGN ACCEPTABLE, WHAT DO WE REALLY MEAN? ARE WE REQUIRING THAT 100% OF THE PRODUCTION (POPULATION) NOT VIOLATE SOME STANDARD WHICH HAS BEEN SET UP? IF THIS IS WHAT WE MEAN BY "ACCEPTABLE", THEN WE CAN FORGET ALL ABOUT SAMPLING PLANS, FOR THE ONLY WAY TO BE CERTAIN THAT 100% OF THE PRODUCTION IS ACCEPTABLE IS TO TEST EVERY PIECE PRODUCED.

LET'S RELAX OUR DEFINITION OF ACCEPTABILITY SO AS TO PERMIT N BAD ITEMS PER YEAR OUT OF ALL T THAT WE PRODUCE. IN ORDER TO BE SURE OF THIS LEVEL OF ACCEPTABILITY, WE MUST EITHER . . .

(A) TEST THE FIRST (T - N) ITEMS AND FIND THAT THEY ALL PASS.

OR

(B) IF WE FIND $K < N$ BAD ONES BEFORE (T - N) HAVE BEEN TESTED, WE CONTINUE TESTING UNTIL WE HAVE (T - N) "ACCEPTABLE" ONES.

IF WE DON'T FIND (T - N) ACCEPTABLE ONES, THE PRODUCT IS RATED UNACCEPTABLE. WHEN _T_ IS THE ANNUAL PRODUCTION TOTAL IN THE MILLIONS, ONE CAN SEE THAT EVEN WITH A RELAXED DEFINITION OF "ACCEPTABILITY" THE SAMPLE SIZE IS PROHIBITIVE. IN OTHER WORDS, DEMANDING 100% CONFIDENCE LEVEL FOR A GIVEN ACCEPTABILITY LEVEL IS TOO IDEALISTIC.

IF WE WANT TO GET INTO THE REALM OF REASONABLE SAMPLING, WE MUST RELAX THE LEVELS OF ACCEPTABILITY AND CONFIDENCE TO BE BOTH BELOW 100%. THE FARTHER WE DROP BELOW THE 100% CONFIDENCE LEVEL, THE SMALLER THE SAMPLE SIZE BECOMES.

TO TAKE A SPECIFIC EXAMPLE WE CAN ASK THE FOLLOWING:

ASSUMING THAT STEERING COLUMNS CAN BE CALLED "ACCEPTABLE" IF AT LEAST 99.9% SHOW A REARWARD DISPLACEMENT LESS THAN 5 INCHES IN A BARRIER CRASH TEST, HOW LARGE A SAMPLE SHOULD BE TESTED IN ORDER TO YIELD 99% CONFIDENCE FOR THIS LEVEL OF ACCEPTABILITY? WHAT ABOUT 95% CONFIDENCE? 90% CONFIDENCE? 75% CONFIDENCE?

SOLUTION

SUPPOSE N (UNKNOWN AS YET) CARS ARE TESTED AND THEY YIELD A MEAN OF 2.225 INCHES OF DISPLACEMENT AND A STANDARD DEVIATION OF .205 INCHES. THE QUESTION IS:

"HOW LARGE MUST N BE IN ORDER TO GUARANTEE 99.9% IN COMPLIANCE WITH THE 5 INCH MAXIMUM WITH ...

(A) CONFIDENCE = 99%?

(B) CONFIDENCE = 95%?

(C) CONFIDENCE = 90%?

(D) CONFIDENCE = 75%?

THE MATHEMATICAL FORMULAS FOR THIS PROBLEM YIELD THE FOLLOWING ANSWERS:

CONFIDENCE LEVEL	REQUIRED SAMPLE SIZE
99%	5
95%	3
90%	2
75%	1

THE FOLLOWING OBSERVATIONS ARE FORTHCOMING:

(1) THE ABOVE ANSWERS ARE VALID ONLY FOR A SAMPLE MEAN OF 2.225 AND A STANDARD DEVIATION OF .205.

(2) THE REQUIRED SAMPLE SIZE ASSUMES ALL TESTED ITEMS ARE ACCEPTABLE.

(3) THE SOLUTION IS ITERATIVE IN NATURE.

NOW WE OBSERVE FROM THE ABOVE TABLE THAT 99% CONFIDENCE CARRIES A PRICE TAG OF 5 TEST CARS. ARE WE WILLING TO PAY THIS PRICE? IF NOT, THEN WE SELECT SOME LOWER CONFIDENCE LEVEL WITH WHICH WE ARE WILLING TO LIVE WITH. IN MOST CASES, THE CONFIDENCE BANDS ARRIVED AT ARE DIFFICULT TO RELATE BOTH TO THE REAL WORLD.

THE EXTENDED APPROACH ESTABLISHES THE OPTIMUM CONFIDENCE LEVEL AND SAMPLE SIZE BY MINIMIZING THE RISK ASSOCIATED WITH A PARTICULAR DESIGN. IN OTHER WORDS, FOR THE ANSWERS TO THE COST OF TESTING AND COST OF POTENTIAL LOSS DUE TO RELEASING AN UNACCEPTABLE DESIGN, WE CAN ESTABLISH THE OPTIMUM SAMPLE SIZE AND CONFIDENCE LEVEL SO AS TO MINIMIZE THE EXPECTED DOLLAR LOSS.

EXTENDED SOLUTION

SUPPOSE THE COST PER TEST CAR IS $5,000 AND SUPPOSE THAT FAILURE TO

PROVIDE AN "ACCEPTABLE" DESIGN (LESS THAN 99.9% PASSING) WOULD CAUSE A $1,000,000 FAILURE EXPENSE. THEN WE WOULD HAVE THE FOLLOWING TABLE OF OUTCOMES:

"ACCEPTABLE" DESIGN	"UNACCEPTABLE" DESIGN
CONFIDENCE = p	CONFIDENCE = 1 - p
FAILURE EXPENSE = $0	FAILURE EXPENSE = V
TEST EXPENSE = NK	TEST EXPENSE = NK
LOSS = 0 + NK	LOSS = V + NK

WHERE:

K = COST PER TEST CAR = $5,000

V = $1,000,000

THE METHOD WILL ESTABLISH THE CONFIDENCE LEVEL p AND THE SAMPLE SIZE N. THE ASSOCIATED RISK WITH THE ABOVE IS

$$RISK = NK + V\ (1 - p)$$

THIS FUNCTION WAS TABULATED AND THE RESULTS FOLLOW.

SAMPLE SIZE	RISK	CONFIDENCE LEVEL
.	.	.
.	.	.
.	.	.
4	41784	97.82%
5	35978	98.90%
6	35730	99.42%
7	38076	99.69%
8	41689	99.83%
.	.	.
.	.	.
.	.	.

THUS, WE HAVE ARRIVED AT AN OPTIMUM SAMPLE SIZE OF 6 WITH ITS CORRESPONDING CONFIDENCE LEVEL OF 99.42% THAT MINIMIZES THE RISK ASSOCIATED WITH THIS DESIGN.

IT SHOULD BE NOTED THAT NO UNIVERSAL SOLUTION CAN BE APPLIED TO ALL SAMPLE SIZE PROBLEMS. IN BRIEF, THE SIZE OF THE SAMPLE DEPENDS UPON WHAT INFORMATION ONE ALREADY HAS AND WHAT KIND OF ADDITIONAL INFORMATION ONE NEEDS. DETROIT RESEARCH INSTITUTE WILL BE HAPPY TO AID EVERYBODY IN THEIR QUEST TO ESTABLISH OPTIMUM SAMPLE SIZES.