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ESTIMATING THE ENDURANCE LIMIT BY FATIGUE TESTING SPECIMENS AT SEVERAL STRESS LEVELS

The use of several stress levels to estimate the endurance limit can be outlined as follows:

(1) Start with a sample of N_1 specimens at some specific stress S_1 and test them all to failure or to the endurance limit bogey (say, 5 million cycles).

(a) If all survive to the endurance limit bogey increase the stress on the next sample of test specimens.

(b) If all fail before the endurance limit bogey reduce the stress on the next sample of specimens.

(c) If some fail before the endurance limit bogey and some survive, the stress may be either reduced in an attempt to have more survivors, or the stress may be increased in order to get fewer survivors to the endurance limit bogey.

(2) The procedure is completed only when enough stress levels have been used so as to have at least one stress where all fail before the endurance limit bogey, at least one stress level where all survive to the endurance level bogey, and at least one stress level where there are both failures and survivors at the endurance level bogey.

A TYPICAL EXAMPLE

(1) Suppose a test engineer tests 10 specimens at 30,000 PSI and finds the following cycles to failure (rearranged in numerical order):

245,000 cycles (failed)
310,000 cycles (failed)
560,000 cycles (failed)
790,000 cycles (failed)
1,215,000 cycles (failed)
2,300,000 cycles (failed)

5,000,000 cycles (unfailed)
5,000,000 cycles (unfailed)
5,000,000 cycles (unfailed)
5,000,000 cycles (unfailed)

(NOTE: 5,000,000 cycles = Endurance Limit Bogey)

(2) Next, the test engineer selects 8 new specimens and runs them at 40,000 PSI. The results are (in order of cycles run):

6,500 cycles (failed)
8,000 cycles (failed)
9,500 cycles (failed)
9,900 cycles (failed)
19,000 cycles (failed)
40,000 cycles (failed)
75,000 cycles (failed)

5,000,000 cycles (unfailed)

(3) Next, the test engineer selects 12 specimens and runs them at 50,000 PSI with the following results, all failed and arranged in order of cycles run:

987 cycles (failed)
1,120 cycles (failed)
1,430 cycles (failed)
2,105 cycles (failed)
2,610 cycles (failed)
3,007 cycles (failed)
4,560 cycles (failed)
5,340 cycles (failed)
6,060 cycles (failed)
8,590 cycles (failed)
8,700 cycles (failed)
9,870 cycles (failed)

(4) Finally, the test engineer selects 7 new specimens and runs them at 20,000 PSI, and finds that all 7 survive to the endurance limit bogey of 5,000,000 cycles.

ANALYSIS OF THE DATA

In this particular example we have the following results (starting with the lowest stress used and proceeding to the highest stress used):

STRESS LEVEL	NUMBER FAILED AT ENDURANCE LIMIT BOGEY
20,000 PSI	0 failed out of 7
30,000 PSI	6 failed out of 10
40,000 PSI	7 failed out of 8
50,000 PSI	12 failed out of 12

We treat any situation with survivors at the endurance limit bogey as attribute data. This would include the first three stress levels above, i.e., 20,000 PSI, 30,000 PSI, and 40,000 PSI. At the highest stress (50,000 PSI) we assume 100% failed.

For 20,000 PSI we have (0 failed out of 7), which is converted into a MEDIAN RANK by adding 1 to the 0 defectives, and by adding 1 to the sample size 7, thus arriving at the MEDIAN RANK of the 1st in 8, which is, by BENARD'S FORMULA,

$$\frac{1 - .3}{8 + .4} = \frac{.7}{8.4} = .083 = 8.3\%$$

For 30,000 PSI we have (6 failed out of 10) which is converted to the MEDIAN RANK of the 7th in 11 (adding 1 to both 6 and 10). Thus, the MEDIAN RANK in this case becomes,

$$\frac{7 - .3}{11 + .4} = \frac{6.7}{11.4} = .588 = 58.8\%$$

For 40,000 PSI we have (7 failed out of 8), which is converted to the MEDIAN RANK of the 8th in 9 (adding 1 to both 7 and 8). Thus, the MEDIAN RANK for this becomes,

$$\frac{8 - .3}{9 + .4} = \frac{7.7}{9.4} = .82 = 82\%$$

PLOTTING THE RESULTS OF THE TESTS

To display the results at the different stress levels we use ordinary graph paper with the stress along the abscissa, and the percent failed (Median Rank) at the endurance limit bogey along the ordinate. For the example here considered we have the following abscissas and ordinates:

ABSCISSA (Stress Level)	ORDINATE (Percent Failed At 5,000,000 Cycles i.e., Median Rank)
20,000	8.3%
30,000	58.8%
40,000	82%
50,000	100%

Plotting these abscissas and ordinates we obtain FIGURE 1.

If we extrapolate linearly through the two lowest points (20,000 PSI and 30,000 PSI) we estimate the endurance limit to be about 18,000 PSI (for 0% failed at 5,000,000 cycles).

It will be noticed that the upper end of the graph in FIGURE 1 is an extension of the line segment joining the 30,000 PSI and 40,000 PSI levels, and projects to 100% failed at 47,500 PSI. This is reasonable, since the selected stress of 50,000 PSI probably would not be the smallest stress for 100% failed.

PROJECTION TO THE ENDURANCE LIMIT

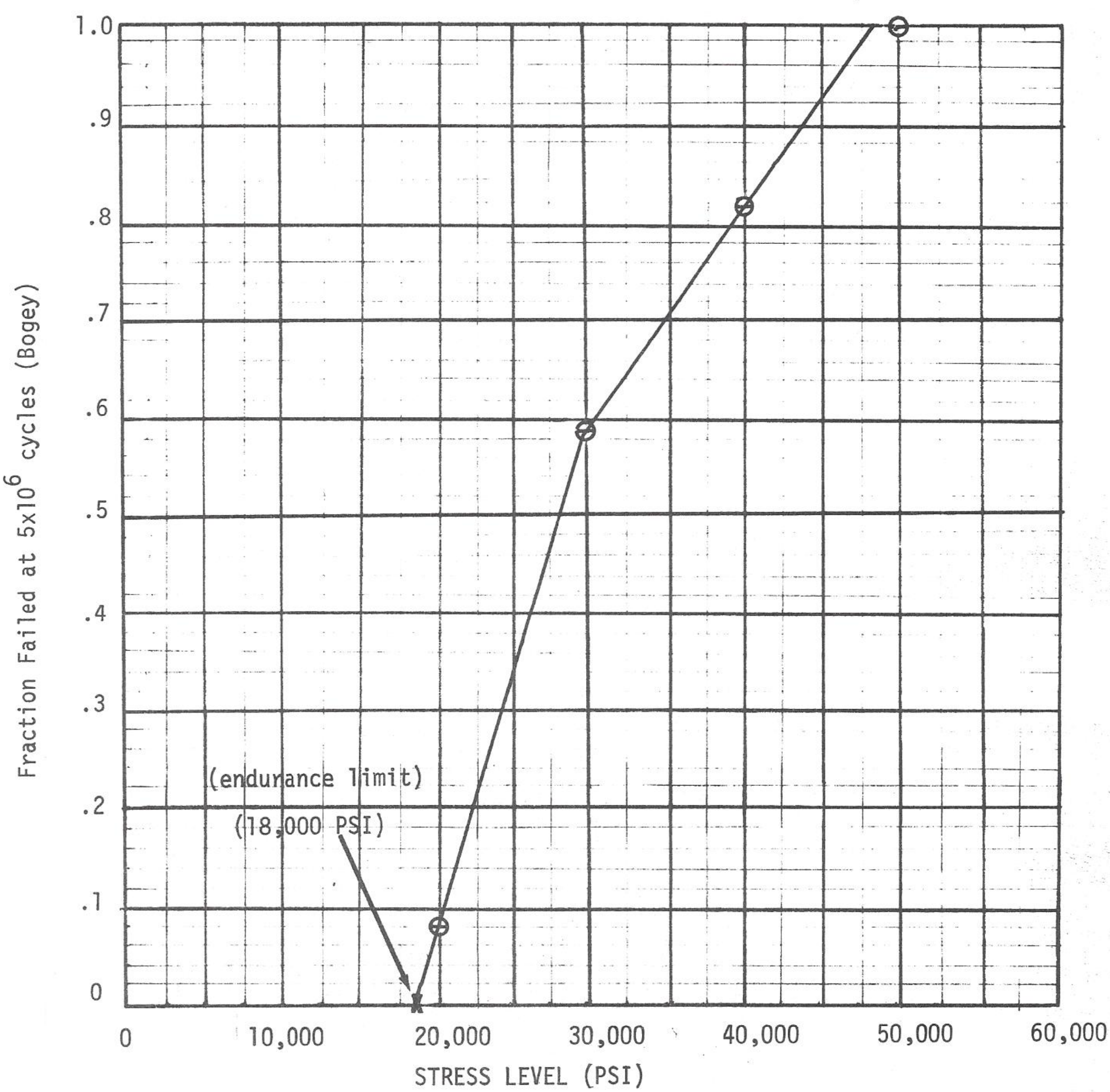

FIGURE 1

A GENERAL FORMULA FOR ESTIMATING THE ENDURANCE LIMIT

Suppose the two lowest stresses used are S_1 (lowest) and S_2 (next to the lowest). Furthermore, suppose at S_1 that there are (F_1 failed out of N_1), and that at S_2 there are (F_2 failed out of N_2).

Now let x = general stress level (abscissa)

and y = fraction failed at bogey (ordinate)

The equation of the straight line which we construct so as to pass through the two lowest stress levels is then,

$$y - \lambda\,(F_1 + 1, N_1 + 1) = (X - S_1)\left[\frac{\lambda(F_2 + 1,\; N_2 + 1) - \lambda(F_1 + 1,\; N_1 + 1)}{S_2 - S_1}\right]$$

where $\lambda\,(A, B)$ = MEDIAN RANK OF THE $A^{\underline{th}}$ in B

$$\simeq \frac{A - .3}{B + .4} \qquad \text{(BENARD'S FORMULA)}$$

Thus,

$$y - \frac{F_1 + 1 - .3}{N_1 + 1 + .4} = \left[\frac{\dfrac{F_2 + 1 - .3}{N_2 + 1 + .4} - \dfrac{F_1 + 1 - .3}{N_1 + 1 + .4}}{S_2 - S_1} \right] (X - S_1)$$

or,

$$y - \frac{F_1 + .7}{N_1 + 1.4} = \left[\frac{\dfrac{F_2 + .7}{N_2 + 1.4} - \dfrac{F_1 + .7}{N_1 + 1.4}}{S_2 - S_1} \right] (X - S_1)$$

Solving this for x yields,

$$x = S_1 + \frac{(S_2 - S_1)\left(y - \dfrac{F_1 + .7}{N_1 + 1.4} \right)}{\dfrac{F_2 + .7}{N_2 + 1.4} - \dfrac{F_1 + .7}{N_1 + 1.4}}$$

To find the endurance limit E, put y = 0, thus,

$$E = S_1 - \frac{\dfrac{(F_1 + .7)\,(S_2 - S_1)}{N_1 + 1.4}}{\dfrac{F_2 + .7}{N_2 + 1.4} - \dfrac{F_1 + .7}{N_1 + 1.4}}$$

or

$$E = S_1 - \frac{S_2 - S_1}{\left(\dfrac{N_1 + 1.4}{N_2 + 1.4}\right)\left(\dfrac{F_2 + .7}{F_1 + .7}\right) - 1};$$

(General Formula for Endurance Limit (50% Confidence))

APPLYING THE GENERAL FORMULA TO THE EXAMPLE REPRESENTED BY FIGURE 1

We have

$$S_1 = 20{,}000 \quad ; \quad N_1 = 7 \quad ; \quad F_1 = 0$$

$$S_2 = 30{,}000 \quad ; \quad N_2 = 10 \quad ; \quad F_2 = 6$$

Therefore,

$$E = 20{,}000 - \frac{30{,}000 - 20{,}000}{\left(\frac{8.4}{11.4}\right)\left(\frac{6.7}{0.7}\right) - 1}$$

$$= 20{,}000 - \frac{10{,}000}{6.05263}$$

$$= 20{,}000 - 1652$$

$$= 18{,}348 \text{ PSI} .$$

ESTIMATING THE ENDURANCE LIMIT WITH ANY DESIRED CONFIDENCE

As we stated alongside the GENERAL FORMULA FOR ENDURANCE LIMIT, the confidence level for the estimate was 50% (see page 9).

Now suppose it is required that the conficence be much higher, say 95%. What would be the appropriately lower estimate of E in such a case? The key to the problem is to substitute 95% ranks wherever we employed median ranks. Doing this will yield the following formula:

$$E_{95\%\ \text{confidence}} = S_1 - \frac{S_2 - S_1}{\left(\dfrac{95\%\ \text{Rank at}\ S_2}{95\%\ \text{Rank at}\ S_1}\right) - 1}$$

In general, for confidence C, the formula for the endurance limit estimate is,

$$E_{\text{conf. C}} = S_1 - \frac{S_2 - S_1}{\left(\dfrac{\text{C-Rank at}\ S_2}{\text{C-Rank at}\ S_1}\right) - 1}$$

95% CONFIDENCE LEVEL OF THE ENDURANCE LIMIT FOR THE NUMERICAL EXAMPLE

At S_1 = 20,000 PSI : 95% Rank of 1^{st} in 8 = .31234

At S_2 = 30,000 PSI : 95% Rank of 7^{th} in 11 = .80042

$$E_{95\%\ \text{confidence}} = 20{,}000 - \frac{30{,}000 - 20{,}000}{\left(\frac{.80042}{.31234}\right) - 1}$$

$$= 20{,}000 - \frac{10{,}000}{1.562656}$$

$$= 20{,}000 - 6400$$

$$= 13{,}600 \text{ PSI}.$$

Thus, there is 95% confidence that the true endurance limit is at least 13,600 PSI.