Statistical Bulletin
Reliability & Variation Research

DETROIT RESEARCH INSTITUTE
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Vol. 1 February, 1972

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DESIGNING ACCELERATED FATIGUE TESTS

INTRODUCTION

The evaluation of the endurance qualities of a new mechanical design can be an aggravatingly expensive and time consuming project. This is especially true of those mechanical parts of a motor vehicle which are never supposed to fail in the field, such as steering knuckles and other items directly related to car control and safety. Since the service life of such items is designed to be very long (theoretically infinite), how can their actual durabilities ever be accurately determined at nominal loads? Admittedly the answer to this question is that it is very difficult, if not impossible, to find a sufficiently large sample of failures to derive an accurate statistical distribution function of miles to failure in service for such items. This being the case, what alternative, if any, exists for durability evaluation? The answer to this last question is found in the design of accelerated fatigue tests. Strict idealists in the theory of fatigue shy away from decision making based on accelerated tests, but practical minded car builders have no such retreat--they must design accelerated fatigue tests to evaluate their designs for durability and safety. The subsequent pages show how such test programs can be designed in a rational fashion.

BASIC TRADE-OFF PRINCIPLES IN FATIGUE TESTING

It should be thoroughly understood that there are certain basic principles which serve as guidelines for designing accelerated fatigue tests. The two most important principles can be stated in terms of mechanical and statistical concepts as follows:

PRINCIPLE #1: If the predicted fraction of specimens surviving x_0 cycles is R_0 at some stress level, and all specimens are run to x_1 cycles at the same stress, then R_1, the new predicted fraction of specimens surviving to x_1 cycles, is given by the formula

$$R_1 = R_0^{\left(\frac{x_1}{x_0}\right)^b}$$

where b= Weibull slope of the life distribution at the test stress.

EXAMPLE USING PRINCIPLE #1

Suppose 90% of a population of some auto component of Weibull slope 2 survives 1,000,000 cycles at 100,000 PSI.

(1) What percent will survive 2,000,000 cycles at the same stress?

(2) What percent will survive 500,000 cycles?

SOLUTION TO (1) :

$b = 2;\quad R_0 = .9;\quad x_0 = 1{,}000{,}000;\quad x_1 = 2{,}000{,}000$

$$\therefore R_1 = .9^{\left(\frac{2{,}000{,}000}{1{,}000{,}000}\right)^2} = .9^{2^2} = .9^4 = 65.61\%\ \text{(ANS.)}$$

SOLUTION TO (2) :

$b = 2;\quad R_0 = .9;\quad x_0 = 1{,}000{,}000;\quad x_1 = 500{,}000$

$$\therefore R_1 = .9^{\left(\frac{500{,}000}{1{,}000{,}000}\right)^2} = .9^{.5^2} = .9^{.25} = .974$$

$= 97.4\%$ (ANS.)

PRINCIPLE #2: If the predicted fraction of specimens surviving x_0 cycles at stress σ_o is R_0, then the predicted fraction of specimens which will survive x_0 cycles at another stress σ_1 is given by the formula

$$R_1 = R_0^{\left(\frac{\sigma_1}{\sigma_o}\right)^{mb}}$$

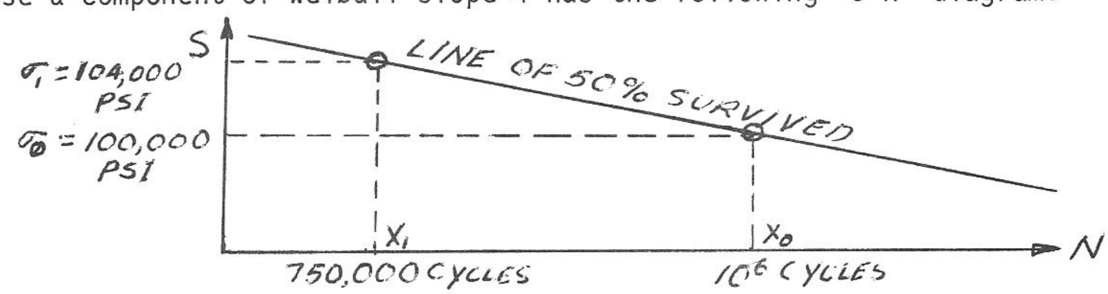
where b = Weibull slope of the life distributions at both stress levels (assumed the same at both stresses),

and m = the average slope of the "S-N" curve on log-log paper between the two stress levels.

(NOTE: It is assumed that σ_1 is not too different from σ_o, say, within ± 20% of σ_o. For any drastic differences between σ_o and σ_1 the Weibull slope could not be assumed constant.)

EXAMPLE USING PRINCIPLE #2

Suppose a component of Weibull slope 4 has the following "S-N" diagram:



This diagram tells us that 50% of the population of this component will survive 1,000,000 cycles at 100,000 PSI. Suppose the stress is raised to 104,000 PSI. What fraction would survive 1,000,000 cycles at this higher stress?

SOLUTION

$b = 4$ σ_o = 100,000 PSI

$R_0 = .5$ σ_1 = 104,000 PSI

$$m = \frac{\text{Log } 1{,}000{,}000 - \text{Log } 750{,}000}{\text{Log } 104{,}000 - \text{Log } 100{,}000} = 7.336$$

(NOTE: THE GENERAL FORMULA FOR m is $m = \dfrac{\text{Log } x_0 - \text{Log } x_1}{\text{Log } \sigma_1 - \text{Log } \sigma_o}$)

$$\therefore \quad R_1 = .5^{\left(\frac{104{,}000}{100{,}000}\right)^{7.336 \times 4}} = .5^{1.04^{29.344}} = .5^{3.161} = .1118$$

$= 11.18\%$ (ANS.)

DESIGNING ACCELERATED FATIGUE TESTS USING THE TWO PRINCIPLES

A good number of problems arising in the design of accelerated fatigue tests of mechanical parts can be solved in a practical fashion by using the two basic principles which we have just presented. Some of the typical situations to which answers can be found are discussed below.

SITUATION #1: WHEN TEST SPECIMENS ARE EXPENSIVE OR IN SHORT SUPPLY, AND WE WANT TO RUN AS FEW AS POSSIBLE, BUT FOR MORE CYCLES EACH, IN ORDER TO GET A GOOD STATISTICAL REPRESENTATION OF PRODUCT RELIABILITY.

For example, suppose a reliability specification requires that 100 test specimens must all survive 1,000,000 cycles at stress 120,000 PSI. Suppose that only 20 such specimens can be supplied. How many cycles must these 20 specimens be run without failure at stress 120,000 PSI, if the Weibull slope is 8?

SOLUTION

We first estimate the fraction of the specimen population which survives 1,000,000 cycles at 120,000 PSI. This is done by calculating

$$R_o = \frac{S + .7}{N + 1.4} \qquad \begin{pmatrix} S = \text{No. of survivors} = 100 \\ N = \text{No. of trials} = 100 \end{pmatrix}$$

$$= \frac{100 + .7}{100 + 1.4} = .9930966$$

Then by PRINCIPLE #1:

$$R_1 = R_o^{\left(\frac{x_1}{x_o}\right)^b} \qquad \begin{pmatrix} x_1 = \text{New Target} \\ x_o = \text{Old Target} = 1{,}000{,}000 \text{ cycles} \end{pmatrix}$$

Furthermore, if 20 survive to the new target x_1, the estimated fraction of the population surviving to the new target is $R_1 = \frac{20 + .7}{20 + 1.4} = .9672897$

Thus by PRINCIPLE #1, $.9672897 = (.9930966)^{\left(\frac{x_1}{1,000,000}\right)^8}$

Solving this for x_1 gives $\left(\frac{x_1}{1,000,000}\right)^8 = \frac{\text{Log } .9672897}{\text{Log } .9930966} = 4.8$

$\therefore \frac{x_1}{1,000,000} = (4.8)^{1/8} = 1.216620$, or $x_1 = 1,216,620$ cycles

Thus, running 20 specimens of Weibull slope 8 successfully for 1,216,620 cycles at 120,000 PSI would be a performance equivalent to 100 specimens of Weibull slope 8 running successfully for 1,000,000 cycles at 120,000 PSI. It is quite a saving in test time and specimen expense, as well!

SITUATION #2: WHEN TESTING TIME AND SAMPLE SIZE CAN BE REDUCED BY INCREASING THE STRESS LEVEL.

For example, suppose a reliability specification calls for testing 200 specimens of Weibull slope 3, and that all 200 must survive 500,000 cycles at a stress of 150,000 PSI. If we increase the stress to 160,000 PSI, how many specimens must survive 500,000 cycles at this increased stress if the "S-N" slope between the two stress levels is m = 7?

SOLUTION

$R_o = \frac{200 + .7}{200 + 1.4} = .9965243$ = Estimated fraction surviving 500,000 cycles at 150,000 PSI

$b = 3$; $m = 7$; $\sigma_0 = 150,000$; $\sigma_1 = 160,000$

According to PRINCIPLE #2: $R_1 = R_0\left(\frac{\sigma_1}{\sigma_0}\right)^{mb}$

Thus, $R_1 = (.9965243)^{\left(\frac{160,000}{150,000}\right)^{21}} = (.9965243)^{3.8779594} = .98659$

Let S_1 = New No. of specimens which must survive 500,000 cycles at the new stress of 160,000 PSI.

Then, $\frac{S_1 + .7}{S_1 + 1.4} = .98659 = R_1$

This last equation reduces to $S_1 + .7 = .98659\ S_1 + 1.381226$

or $.01341\ S_1 = .681226$

$S_1 = 50.8$ or 51 (To next integer)

Thus, 51 specimens of Weibull slope 3 surviving 500,000 cycles at 160,000 PSI is a performance which is equivalent to 200 specimens of Weibull slope 3 surviving 500,000 cycles at 150,000 PSI when the average "S-N" slope between the two stress levels is m = 7.

SITUATION #3: WHEN WE WISH TO COMPARE A NEW SET OF FATIGUE TEST RESULTS TO SOME BOGEY SPECIFICATIONS.

EXAMPLE

Suppose we have tested 10 specimens of some auto component to failure at 106,000 PSI, and obtained a Weibull plot whose slope is 6. Furthermore, suppose the Weibull plot shows 20% failed at 1,500,000 cycles. A bogey specification requires that 95% of such components should survive 5×10^6 cycles when stressed at 90,000 PSI. On the basis of the Weibull plot of 10 specimens can we conclude that the auto part passes the bogey specification, if we know the "S-N" slope between 90,000 PSI and 106,000 PSI is $m = 9$?

SOLUTION

By combining PRINCIPLE #2 with PRINCIPLE #1, we arrive at the basic equation for the actual R_1 as

$$R_1 = R_0^{\left(\frac{x_1}{x_0}\right)^b \left(\frac{\sigma_1}{\sigma_0}\right)^{mb}}$$

where, in this case, $R_0 = .8$ (Since 20% failed at 1,500,000 cycles and stress 106,000 PSI)

$x_0 = 1{,}500{,}000$ cycles; $\sigma_0 = 106{,}000$ PSI; $b = 6$

$x_1 = 5 \times 10^6$ cycles; $\sigma_1 = 90{,}000$ PSI; $m = 9$

Thus, the actual R_1 is estimated to be

$$R_1 = (.8)^{\left[\frac{(5 \times 10^6)}{(1{,}500{,}000)}\right]^6 \left[\frac{(90{,}000)}{(106{,}000)}\right]^{54}} = (.8)^{.199451} = .95647$$

Since this estimated actual R_1 exceeds the bogey of 95% for R_1. we conclude that the test results demonstrate that we have an acceptable component.