**Statistical Bulletin**
Reliability & Variation Research

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In this issue of our STATISTICAL BULLETIN we are pleased to announce an important breakthrough! After years of struggling, stumbling, and disappointment we have finally found a practical empirical formula for calculating rank tables for any confidence index desired and for any sample size. This is the first time such a useful formula is being published. It is truly the answer to this man's prayers as well as those of many of his students over the last decade. Now to proceed to the formula, let us define some symbols:

Let $N$ = sample size

Let $j$ = order statistic number

Let $c$ = confidence index (a decimal between 0 and 1)

Let $Z_c\,(j/N)$ = c-rank of the $j^{th}$ order statistic in $N$

Let $Z_{.50}\,(j/N)$ = median rank of $j^{th}$ order statistic in $N = \dfrac{j - .3}{N + .4}$

Define $$A_N(j) = 1 + \frac{.45N^{(6c)}(j - 1)(N - j)}{(N - 1)^2}$$

Define $$\mu = \frac{\text{LOG}\left[1 - c^{\frac{1}{j \cdot A_N(j)}}\right]}{\text{LOG}\left(1 - .5^{\frac{1}{j \cdot A_N(j)}}\right)}$$

Then, the c-rank of the $j^{th}$ order statistic in N is given by the formula

$$Z_c\ (j/N) = 1 - \left[1 - Z_{.50}\ (j/N)\right]^{\mu}$$

It will be found that this formula gives good approximations which cannot be distinguished from the exact values by the naked eye when plotted on Weibull paper. As such, it serves as a useful tool in all computerized systems concerned with lot control problems in reliability programs where the method of variables is employed.

In order to illustrate the use of this formula, let us calculate the 55% rank of the $7^{th}$ order statistic in 20.

We have: $j = 7$

$N = 20$

$c = .55$

$$Z_{.50}\ (7/20) = \frac{7 - .3}{20 + .4} = \frac{6.7}{20.4} = .3284$$

$$A_N(j) = 1 + \frac{.45 \times 20^{(.33)} (7 - 1)(20 - 7)}{(20 - 1)^2}$$

$$= 1 + \frac{.45 \times 2.6874 (6)(13)}{361}$$

$$= 1.2613$$

$$jA_N(j) = 7^{1.2613} = 11.64$$

$$\frac{1}{jA_N(j)} = \frac{1}{11.64} = .08592$$

$$\mu = \frac{\text{LOG}(1 - .55^{.08592})}{\text{LOG}(1 - .5^{.08592})} = \frac{\text{LOG}(1 - .9499)}{\text{LOG}(1 - .9422)}$$

$$= \frac{\text{LOG}(.0501)}{\text{LOG}(.0578)} = \frac{-2 + .69984}{-2 + .76193} = \frac{-1.30016}{-1.23807} = 1.05015$$

$$\therefore Z_{.55}(7/20) = 1 - (1 - .3284)^{1.05015} = 1 - .6716^{1.05015}$$

$$= 1 - .6583 = .3417$$

The correct value of $Z_{.55}$ (7/20) to four decimal places is .3411, so we are off only by .0006 in our approximation for this case!

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## THE LOGIC OF CONFIDENCE SUPERPOSITION FOR AN ASSEMBLY RELIABILITY FOR TWO COMPONENTS IN SERIES

Given: Component #1 with $R_{C_1}(X_o) = R_1$

Component #2 with $R_{C_2}(X_o) = R_2$

ASSEMBLY with $R_{\hat{C}}(X_o) = \hat{R}$

where $$\hat{R} = R_1 R_2 \qquad (1)$$

and $$\hat{C} = \frac{C_1 C_2}{C_1 C_2 + (1 - C_1)(1 - C_2)} \qquad (2)$$

CONFIDENCE [That Component #1 beats $R_1$] = $C_1$

CONFIDENCE [That Component #2 beats $R_2$] = $C_2$

Thus, we have two evidences* that samples tested beat their standard baselines, i.e., a sample of component #1 which is ahead of its baseline $R_1$ with confidence $C_1$ for an evidence $EV_1 = \ln\left(\frac{C_1}{1 - C_1}\right)$, and a sample of component #2 which is ahead of its baseline $R_2$ with confidence $C_2$ for an evidence $EV_2 = \ln\left(\frac{C_2}{1 - C_2}\right)$. The sum of these two evidences is the total evidence that the system beats its baseline $\hat{R} = R_1 \cdot R_2$, i.e., if $\hat{C}$ = RESULTANT CONFIDENCE,

$$\text{then} \quad \ln\left(\frac{\hat{C}}{1 - \hat{C}}\right) = \ln\left(\frac{C_1}{1 - C_1}\right) + \ln\left(\frac{C_2}{1 - C_2}\right),$$

which yields relation (2) on the previous page.

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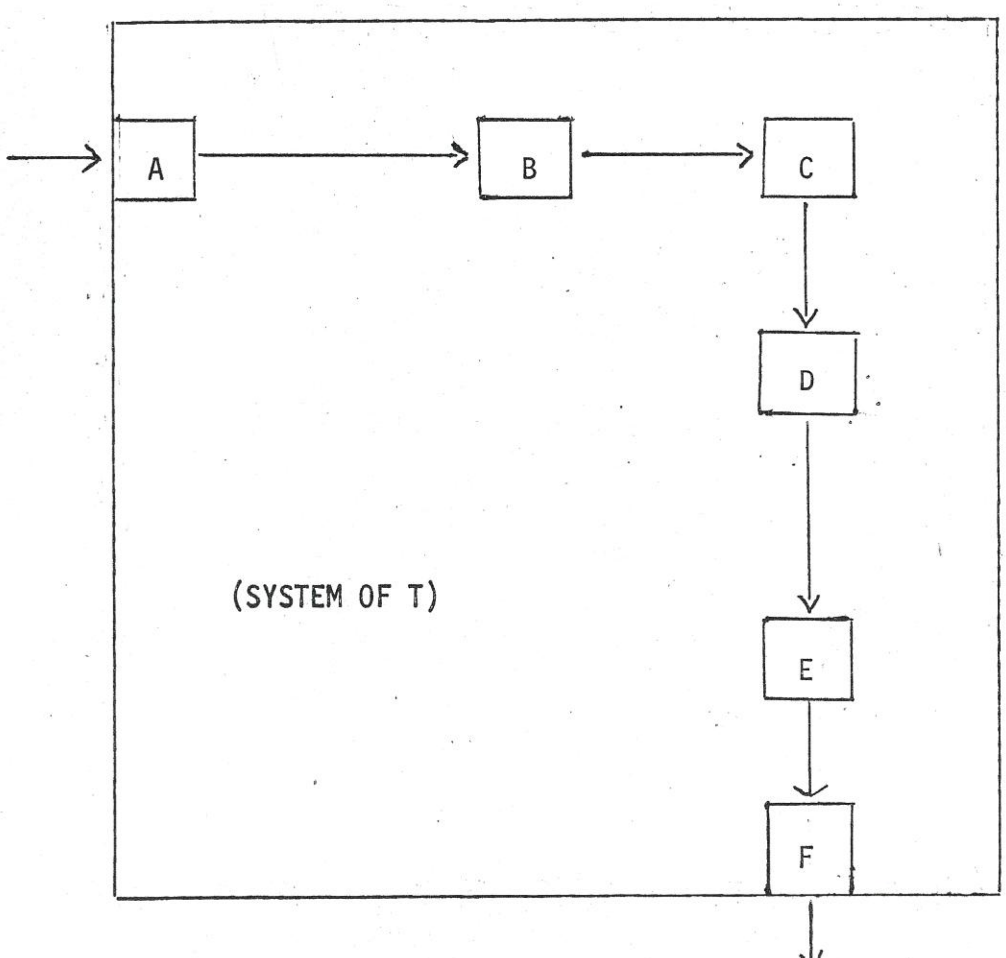
*Evidence as here used is defined to be the natural logarithm of the odds ratio, where

$$\text{ODDS RATIO} = \frac{\text{CONFIDENCE}}{1 - \text{CONFIDENCE}} .$$

USING ENTROPIES TO CALCULATE THE RELIABILITY OF
A SYSTEM CONSISTING OF COMPONENTS IN SERIES

Suppose we have the following system T:




This system T consists of six components in series (A,B,C,D,E,F).

PROBLEM: Predict the system reliability for a target of 1000 hours.

Let us suppose these six components have Weibull life distributions:

Let $\alpha$ = Minimum life

$b$ = Weibull slope

$\theta$ = Characteristic life

Construct the following table:

| (1) COMPONENT | (2) $\alpha$ | (3) b | (4) $\theta$ | (5) ENTROPY AT 1000 HRS. | (6) RELIABILITY (1000 HRS.) |
|---|---|---|---|---|---|
| A | 100 | 1 | $10^6$ | .000900090 | |
| B | 0 | 2 | $10^5$ | .000100000 | |
| C | 0 | 3 | $2x10^5$ | .000000125 | |
| D | 200 | 1 | $10^5$ | .008016032 | |
| E | 0 | 2 | $10^6$ | .000001000 | |
| F | 0 | 2 | $10^4$ | .010000000 | |
| SYSTEM T | | | | TOTAL ENTROPY .019017247 | .98098 |

Columns (2), (3), and (4) give the Weibull parameters for the components in the system.

Column (5) (Entropy at 1000 hrs.) is completed by calculating the entropy for each component at 1000 hrs. as follows:

$$\text{ENTROPY AT 1000 HRS.} = \left( \frac{1000 - \alpha}{\theta - \alpha} \right)^{b}$$

(Using the $\alpha$ , b, and $\theta$ for the component in question)

The total system entropy at 1000 hours is then the <u>sum</u> of the component entropies at 1000 hours.

Let $\hat{E}$ = Total System Entropy (At 1000 hours).

The TOTAL SYSTEM RELIABILITY at 1000 hours (Column (6) ) is then

$$\hat{R} = e^{-\hat{E}} = e^{-.019017} = .98098 .$$

This turns out to be a very simple procedure for computerizing the calculation of such a system reliability.