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THE THEORY OF OPERATIONAL MAINTAINABILITY

FACTORS WHICH MUST BE CONSIDERED IN PROBLEMS OF PRODUCT MAINTAINABILITY

TIME FACTORS

(1) The time to a failure (unscheduled repair)

(2) The time to a scheduled repair (overhaul)

(3) The time required to make a repair (down time)

RATE FACTORS

(4) Failures per week (unscheduled repair per week)

(5) Number of repairs which can be made per week

(6) Scheduled repairs per week

(7) Unscheduled repairs per week

SIZE FACTORS

(8) Number of machines in the operation

(9) Number of repairmen

(10) Number of inoperable machines (length of repair queue)

MACHINE TYPE	NO. OF MACHINES	WEIBULL PARAMETERS
1	N_1	$(\alpha_1, b_1, \theta_1)$
2	N_2	$(\alpha_2, b_2, \theta_2)$
.	.	.
.	.	.
.	.	.
i	N_i	$(\alpha_i, b_i, \theta_i)$
.	.	.
.	.	.
.	.	.
k	N_k	$(\alpha_k, b_k, \theta_k)$

α_i = Minimum Life of each machine of Type i

b_i = Weibull Slope of the Failure Distribution of each machine of Type i

θ_i = Characteristic Life of each machine of Type i

MEAN NO. OF UNSCHEDULED REPAIRS NEEDED PER WEEK
(i.e., Mean No. of Failures Per Week)

Mean No. of Failures Per Week of 40 Hours :

$$N_1\left(\frac{40-\alpha_1}{\theta_1-\alpha_1}\right)^{b_1}+N_2\left(\frac{40-\alpha_2}{\theta_2-\alpha_2}\right)^{b_2}+\cdots+N_k\left(\frac{40-\alpha_k}{\theta_k-\alpha_k}\right)^{b_k}=\bar{F}_w$$

WEEKLY REPAIR CAPABILITY

Let N_R = Number of Repairmen

Let M_t = Mean Down Time

(Simultaneous Repair Rate Capability) = N_R repairs in M_t hours

No. of Repairs which can be made in a 40 hour week = $(40/M_t)$ Repairs Per Week Per Repairman.

No. of Repairs N_R Repairmen can make per week of 40 hours

$= (40\ N_R/M_t)$ Repairs Per Week

Denote this weekly repair capability by the symbol W_R.

Thus,

$$W_R = \frac{40\ N_R}{M_t}$$

THE NUMBER OF UNREPAIRED MACHINES EACH WEEK

Since the expected number of failures each week is $\bar{F}_W$, and since we are capable of repairing W_R machines each week, it follows that the QUEUE of unrepaired machines grows at a rate equal to

$$\Delta_W = (\bar{F}_W - W_R) \text{ Machines / Week} \qquad (\text{if } \bar{F}_W > W_R)$$

$$\Delta_W = 0 \qquad (\text{if } \bar{F}_W < W_R)$$

THE EFFECT OF SCHEDULED RENEWALS (OVERHAULS) - ALL MACHINES-

If each machine is completely renewed every x_o hours, then the weekly repair total (both scheduled and unscheduled repairs) will average out to

$$\bar{G}_W = \left(\frac{40}{x_o}\right)\sum_{i=1}^{k} N_i\left(\frac{x_o-\alpha_i}{\theta_i-\alpha_i}\right)^{b_i} + \left(\frac{40}{x_o}\right)\sum N_i$$

or

$$\bar{G}_W = \left(\frac{40}{x_o}\right)\left\{\sum_{i=1}^{k} N_i\left[1 + \left(\frac{x_o-\alpha_i}{\theta_i-\alpha_i}\right)^{b_i}\right]\right\}$$

Where $(40/x_o)$ = the greatest integer in $40/x_o$

In case 40 is not exactly divisible by x_o then to $\bar{G}_W$ must be added the additional amount

$$\sum_{i=1}^{k} N_i\left(\frac{x_r-\alpha_i}{\theta_i-\alpha_i}\right)^{b_i}$$

Where x_r = the remainder after dividing 40 by x_o.

THE QUEUE OF UNREPAIRED MACHINES WHEN RENEWALS ARE SCHEDULED EVERY x_o HOURS (ON ALL MACHINES)

Each week the total number of unrepaired machines increase at the rate

$$\delta_W = (\bar{G}_W - W_R) \quad \text{machines per week} \quad (\text{if } \bar{G}_W > W_R)$$

$$\delta_W = 0 \quad (\text{if } \bar{G}_W < W_R)$$

THE IDEAL RENEWAL SCHEDULE

The ideal renewal time x_o is such that

$$\bar{G}_W = W_R$$

$$\therefore \left(\frac{40}{x_o}\right) \sum_{i=1}^{k} N_i \left[1 + \left(\frac{x_o - \alpha_i}{\theta_o - \alpha_i} \right)^{\ell_i} \right] = \frac{40 N_R}{M_t}$$

$$\text{or} \quad \sum_{i=1}^{k} N_i \left[1 + \left(\frac{x_o - \alpha_i}{\theta_o - \alpha_i} \right)^{\ell_i} \right] = \frac{N_R \, x_o}{M_t}$$

To find the ideal renewal period solve this equation for x_o .

EXAMPLE OF HOW A REPAIR BACKLOG BUILDS UP AND ACCELERATES WHEN THERE IS NO SCHEDULED MAINTENANCE

EXAMPLE

A plant has 500 machines. These machines have a Characteristic Life of 1000 hours and a Weibull Slope of 2 . There are 2 Repairmen on the payroll. The Mean Down Time for a machine failure is 8 hours. How large a backlog of unrepaired machines will be accumulated in 16 weeks ?

WEEK NO. (i)	AVE. CUMULATIVE FAILURES	AVE FAILURES DURING THE WEEK	REPAIR CAPABILITY	REPAIR QUEUE
1	0.8	0.8	10	0
2	3.2	2.4	10	0
3	7.2	4.0	10	0
4	12.8	5.6	10	0
5	20.0	7.2	10	0
6	28.8	8.8	10	0
7	39.2	10.4	10	0.4
8	51.2	12.00	10	2.4
9	64.8	13.6	10	6.0
10	80.0	15.2	10	11.2
11	96.8	16.8	10	18.0
12	115.2	18.4	10	26.4
13	135.2	20.0	10	36.4
14	156.8	21.6	10	48.0
15	180	23.2	10	61.2
16	204.8	24.8	10	76.0

$$N\left(\frac{x}{\theta}\right)^b$$

$$\begin{pmatrix} N = 500 \\ \theta = 1000 \\ b = 2 \\ N_R = 2 \\ M_t = 8 \end{pmatrix}$$

$$W_R = \frac{40N_R}{M_t}$$

NOTE : $x = 40\ i$

QUESTIONS REGARDING THE PRECEDING EXAMPLE

QUESTION #1 : How many repairmen are needed in order to fully provide for all repairs in the first 16 weeks ?

SOLUTION : The weekly repair capability must be 24.8 .

Thus ,

$$W_R = \frac{40\ N_R}{M_t} = 24.8$$

or

$$\frac{40\ N_R}{8} = 24.8$$

or

$$5\ N_R = 24.8 \quad , \quad N_R = 5 \text{ repairmen needed}$$

QUESTION #2 : How many failures will occur in week # 52 ? How many repairmen will be needed then ?

SOLUTION :

$$500\left(\frac{2080}{1000}\right)^2 - 500\left(\frac{2040}{1000}\right)^2 = 82.4$$

Thus , to fully provide for weekly failures for a full year , it will be necessary to have 82.4/5 = 17 repairmen by the end of the year.

(an additional repairman must be hired every 3 weeks)

QUESTION # 3 : For the same Weibull Slope (b = 2) , what should the characteristic life θ be in order that no additional repairmen would be needed in one year ?

SOLUTION : θ must be large enough to make the number of machine failures in week #52 equal to 10 .

Thus ,

$$500\left(\frac{2080}{\theta}\right)^2 - 500\left(\frac{2040}{\theta}\right)^2 = 10$$

$$\frac{2080^2}{\theta^2} - \frac{2040^2}{\theta^2} = \frac{10}{500} = \frac{1}{50}$$

$$\frac{1}{\theta^2}(2080^2 - 2040^2) = 1/50 \quad , \quad 164,800/\theta^2 = 1/50$$

$$\theta^2 = 8,240,000 \qquad \underline{\theta = 2871 \text{ hours}}$$

CHECK :

$$500\left(\frac{2080}{2871}\right)^2 - 500\left(\frac{2040}{2871}\right)^2$$

$$500\,(.5248802 - .5048867)$$

$$500\,(.0199935) = 9.99675 \approx 10 \quad \text{ok.}$$

QUESTION #4 : For the same characteristic life of 1000 hours, why would it be better to have machines with a Weibull Slope of Unity (b = 1) instead of the Weibull Slope two (b = 2) ?

ANSWER

For two good reasons. These are

REASON #1 : Fewer repairmen are needed.

In fact , the number of machine failures per week would be constant and equal to $500\left(\frac{40}{1000}\right)^1 = 20$ failures per week.

This would require 20/5 = 4 repairmen.

REASON #2 : At the end of the year , the machines would still be theoretically like new instead of being all worn out as they are when b = 2.

QUESTION #5 : What Weibull Slope b (assuming θ = 1000 hrs.) will allow us to get by with 8 repairmen for the first year ?

SOLUTION : 8 repairmen can handle 8 x 5 = 40 failures a week. Hence , in week #52 there must not be more than 40 failures. This means that b must be such that

$$500\left(\frac{2080}{1000}\right)^b - 500\left(\frac{2040}{1000}\right)^b = 40$$

$$\delta = 2.08^b - 2.04^b = \frac{40}{500} = \frac{4}{50} = .08$$

b	2.08^b	2.04^b	δ
1.4	2.7880	2.7132	.0748
1.5	2.9998	2.9137	.0861

.08 is at 52/113 of this difference

Try $b = 1.4 + (52/113)(.1) = 1.46$

Then $\delta = 2.08^{1.46} - 2.04^{1.46} = 2.9132 - 2.8318 = .0814$ too high

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Try $b = 1.45$

Then $\delta = 2.08^{1.45} - 2.04^{1.45} = 2.8920 - 2.8117 = .0803$ slightly high

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Try $b = 1.44$

Then $\delta = 2.08^{1.44} - 2.04^{1.44} = 2.8708 - 2.7917 = .0791$ slightly low

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A Weibull Slope of 1.44 will do .

QUESTION #6 : Turn to next page .

QUESTION #7 : In answer to Question #6 we found an average repair total of 161.28 in 16 weeks. Find a 90% confidence interval around the MEAN.

SOLUTION

MEAN = 161.28 Repairs

STANDARD DEVIATION $= \sqrt{161.28} = 12.70$ Repairs

SKEWNESS $= \dfrac{2}{\sqrt{161.28}} = \dfrac{2}{12.70} + .1575$

$$t_{.05} = -\left[1.645 - \left(\frac{.1575}{.2}\right)(.059)\right] = -1.599$$

$$t_{.95} = +\left[1.645 + \left(\frac{.1575}{.2}\right)(.055)\right] = +1.688$$

Therefore , the 90% confidence interval on the total repairs required in 16 weeks is from

$$\left[161.28 - 1.599\,(12.70) \quad \text{to} \quad 161.28 + 1.688\,(12.70)\right]$$

or from (140.97 to 182.72) Repairs in 16 Weeks .

QUESTION #6 : If 2- machines are renewed each weekend, how much will this reduce the failure total in 16 weeks ?

SOLUTION

Week # (n)	Average Failures during the Week		Repair Queue	Ave. Cumulative Failures
1	$500(.04)^2$	= 0.8000	-	0.8000
2	$20(.04)^2 + 480\left[(.08)^2 - (.04)^2\right]$	= 2.3360	-	3.1360
3	$20(.08)^2 + 460\left[(.12)^2 - (.08)^2\right]$	= 3.8080	-	6.9440
4	$20(.12)^2 + 440\left[(.16)^2 - (.12)^2\right]$	= 5.2160	-	12.160
5	$20(.16)^2 + 420\left[(.20)^2 - (.16)^2\right]$	= 6.5600	-	18.720
6	$20(.20)^2 + 400\left[(.24)^2 - (.20)^2\right]$	= 7.840	-	26.560
7	$20(.24)^2 + 380\left[(.28)^2 - (.24)^2\right]$	= 9.056	-	35.616
8	$20(.28)^2 + 360\left[(.32)^2 - (.28)^2\right]$	= 10.208	0.208	45.824
9	$20(.32)^2 + 340\left[(.36)^2 - (.32)^2\right]$	= 11.296	1.504	57.120
10	$20(.36)^2 + 320\left[(.40)^2 - (.36)^2\right]$	= 12.320	3.824	69.440
11	$20(.40)^2 + 300\left[(.44)^2 - (.40)^2\right]$	= 13.280	7.104	82.720
12	$20(.44)^2 + 280\left[(.48)^2 - (.44)^2\right]$	= 14.176	11.280	96.896
13	$20(.48)^2 + 260\left[(.52)^2 - (.48)^2\right]$	= 15.008	16.288	111.904
14	$20(.52)^2 + 240\left[(.56)^2 - (.52)^2\right]$	= 15.776	22.064	127.680
15	$20(.56)^2 + 220\left[(.60)^2 - (.56)^2\right]$	= 16.480	28.544	144.160
16	$20(.60)^2 + 200\left[(.64)^2 - (.60)^2\right]$	= 17.120	35.664	161.280

FOR DIFFERENTIAL METHOD :

$d_1 = +1.536$

$d_2 = -.064$

$T_n = 1.632\,n - .032\,n^2 - .8$

$S_n = \frac{.064n}{6}(1 + 75n - n^2)$