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HOW GAINS AND LOSSES DICTATE SAMPLE SIZES , CONFIDENCE LEVELS, AND PRODUCT RELIABILITY GOALS

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INTRODUCTION

WHEN A MANUFACTURER INTENDS TO PUT A PRODUCT ON THE MARKET, THERE ARE A NUMBER OF DECISIONS TO BE MADE. AMONG THESE ARE DECISIONS CONCERNING RELIABILITY, CONFIDENCE, AND SAMPLE SIZES IN TESTING. WHAT IS FINALLY DECIDED ABOUT ALL THESE FACTORS WILL DEPEND UPON THE FINANCIAL EFFECTS OF HAVING FAILURES IN THE FIELD, AS WELL AS THE GAINS REALIZED FROM SUCCESSES AND SATISFIED CUSTOMERS. WHAT IT BOILS DOWN TO IS THAT EVERY GOOD ITEM GAINS A CERTAIN NUMBER OF DOLLARS ($G), WHILE EVERY BAD ITEM LOSES A CERTAIN NUMBER OF DOLLARS ($L). IN THE LONG RUN, THE MANUFACTURER WANTS THE POSITIVE DOLLARS (GAINS) TO BE SEVERAL TIMES LARGER IN MAGNITUDE THAN THE NEGATIVE DOLLARS (LOSSES). THIS RATIO (CALL IT K), WHICH IS DEFINED BY THE FORMULA

$$K = \frac{\text{LONG TERM GAINS}}{\text{LONG TERM LOSSES}},$$

WILL DICTATE WHAT RELIABILITY, CONFIDENCE, AND TEST SAMPLE SIZE ARE NEEDED IN ANY SITUATION.

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THE BASIC MATHEMATICAL PROBLEM

GIVEN: {EACH GOOD ITEM GAINS \$G; EACH BAD ITEM LOSES \$L}

THE MANUFACTURER WANTS TO END UP WITH LONG TERM GAINS FROM GOOD ITEMS K TIMES AS GREAT AS LONG TERM LOSSES FROM BAD ITEMS.

NOTE: IN THIS DISCUSSION, WE ARE ONLY CONCERNED WITH THE REALLY PROBLEMATICAL CASES, IN WHICH $L > G$.

QUESTION: WHAT IS THE MINIMUM RELIABILITY WHICH MUST BE GUARANTEED, TOGETHER WITH ITS CONFIDENCE LEVEL, AND SUCCESS RUN SAMPLE SIZE?

I : THE MINIMUM RELIABILITY

WE DEFINE THE MINIMUM RELIABILITY WHICH CAN BE TOLERATED AS THAT RELIABILITY AT WHICH THE LONG TERM LOSSES JUST EQUAL THE LONG TERM GAINS.

LET $R_{MIN.}$ = MINIMUM RELIABILITY.

THEN, IF WE SELL A TOTAL OF T ITEMS TO CUSTOMERS, THERE WILL BE $TR_{MIN.}$ GOOD ITEMS AND $T(1-R_{MIN.})$ BAD ITEMS.

THE TOTAL GAIN FROM THE GOOD ITEMS WILL BE $TGR_{MIN.}$ DOLLARS, WHILE THE TOTAL LOSS FROM THE BAD ITEMS WILL BE $TL(1-R_{MIN.})$ DOLLARS. FOR RELIABILITY $R_{MIN.}$, THESE TWO QUANTITIES ARE EQUAL. THUS,

$$TGR_{MIN.} = TL(1-R_{MIN.})$$

$$\text{OR} \quad R_{MIN.} = \frac{L}{L+G} \quad \text{(ANS.)}$$

III: THE SUCCESS RUN SAMPLE SIZE REQUIRED IN TESTING

SINCE WE WANT $\left(\frac{\text{LONG TERM GAINS}}{\text{LONG TERM LOSSES}}\right) = K$, IT FOLLOWS THAT WE MUST RELATE K TO THE BEST ESTIMATE LEVEL, WHICH IS AT 50% CONFIDENCE.

LET N = REQUIRED SUCCESS RUN.

THEN, WITH 50% CONFIDENCE, THE RELIABILITY IS

$$R_{.50} = \frac{N + .7}{N + 1.4} \qquad \text{(USING BENARD'S APPROX.)}$$

THEN, FOR A SALES TOTAL T, THE LONG TERM GAINS ARE $TGR_{.50}$ DOLLARS, WHILE THE LONG TERM LOSSES ARE $TL(1 - R_{.50})$ DOLLARS.

NOW, WE WANT

$$\frac{TGR_{.50}}{TL(1 - R_{.50})} = K$$

OR

$$\frac{GR_{.50}}{L(1 - R_{.50})} = K$$

OR

$$\frac{G\left(\frac{N + .7}{N + 1.4}\right)}{L\left(1 - \frac{N + .7}{N + 1.4}\right)} = K$$

OR

$$\frac{G(N + .7)}{.7L} = K$$

OR

$$N = .7\left(\frac{KL}{G} - 1\right) \qquad \text{(ANS.)}$$

III: THE REQUIRED CONFIDENCE LEVEL FOR R_{MIN}.

IT IS KNOWN THAT FOR A SUCCESS RUN SAMPLE SIZE N, THE RELATION BETWEEN RELIABILITY AND CONFIDENCE IS

$$C = 1 - R^{N+1} \qquad (1)$$

IN THE PROBLEM UNDER CONSIDERATION, WE HAVE

$$R = R_{MIN}.$$

$$\text{AND} \quad N = .7\left(\frac{KL}{G} - 1\right)$$

$$\text{THEREFORE,} \quad N + 1 = .3 + \frac{.7KL}{G}$$

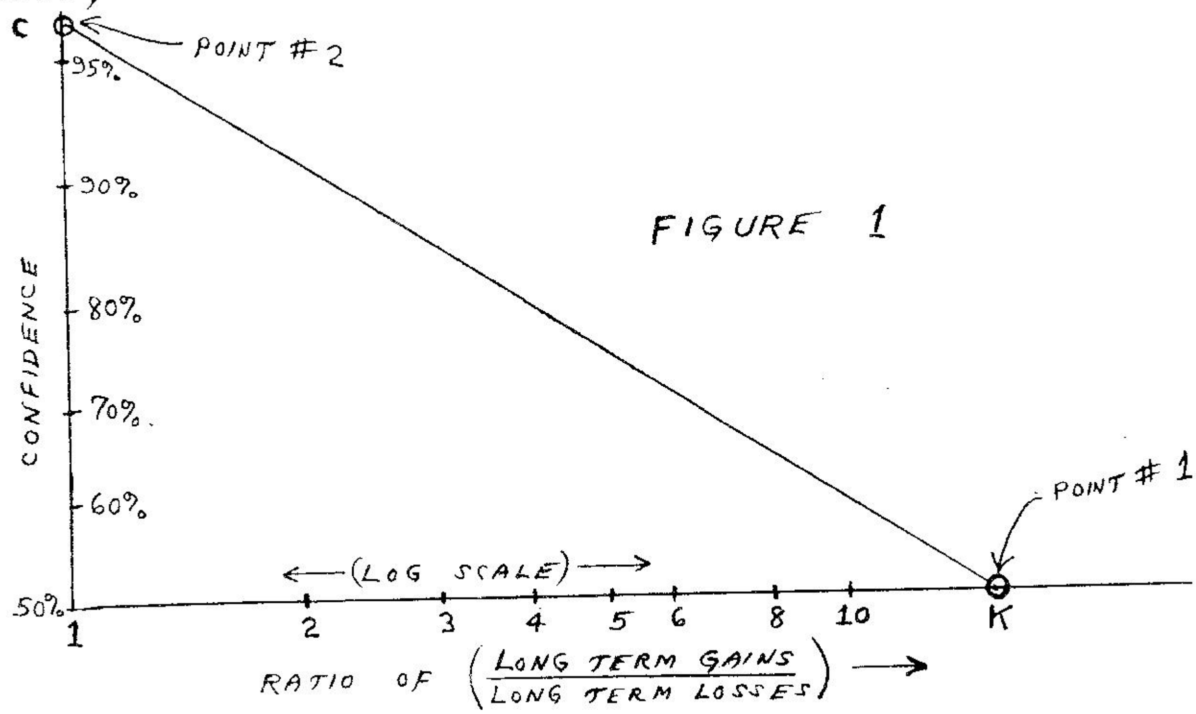
$$\text{FURTHERMORE,} \quad R_{MIN.} = \frac{L}{L + G}$$

THUS, FROM (1), WE OBTAIN THE REQUIRED CONFIDENCE LEVEL:

$$C = 1 - \left(\frac{L}{L + G}\right)^{\left(.3 + \frac{.7KL}{G}\right)} \qquad \text{(ANS.)}$$

IV: GRAPHICAL REPRESENTATION OF THE RESULTS ON LOGARITHMIC CONFIDENCE INTERPOLATION PAPER

WE CAN CONSTRUCT A GRAPHICAL PICTURE OF THE RELATIONS WE HAVE DISCUSSED IN THIS BULLETIN BY CONSTRUCTING A LOGARITHMIC CONFIDENCE INTERPOLATION DIAGRAM, AS SHOWN IN FIGURE 1.



IN THIS LOGARITHMIC CONFIDENCE INTERPOLATION DIAGRAM, POINT #1 HAS ABSCISSA K AND ORDINATE 50%, WHILE POINT #2 HAS ABSCISSA 1 AND AN ORDINATE EQUAL TO THE CONFIDENCE C, WHICH WAS CALCULATED IN SECTION III. WE JOIN POINT#1 WITH POINT #2, USING A STRAIGHT LINE. THEN ALONG THIS LINE, WE CAN READ THE CONFIDENCE FOR ANY DESIRED VALUE OF THE RATIO $\left(\frac{\text{LONG TERM GAINS}}{\text{LONG TERM LOSSES}}\right)$.

V: A NUMERICAL EXAMPLE

SUPPOSE THAT EACH GOOD ITEM GAINS $G = \$100$, AND EACH BAD ITEM LOSES $L = \$1200$.

THEN, THE MINIMUM RELIABILITY WHICH CAN BE TOLERATED IS

$$R_{MIN.} = \frac{L}{L+G} = \frac{1200}{1200+100} = .92308.$$

IF WE WANT $\left(\frac{\text{LONG TERM GAINS}}{\text{LONG TERM LOSSES}}\right) = 10$, THEN THE REQUIRED SUCCESS RUN (FOR $K = 10$) IS

$$N = .7\left(\frac{KL}{G} - 1\right) = .7\left[\frac{(10)(1200)}{100} - 1\right] = 83.3.$$

THUS, TO THE NEXT INTEGER: $N = 84$.

THE CONFIDENCE FOR THE MINIMUM RELIABILITY IS (BY SECTION III):

$$C = 1 - \left(\frac{L}{L+G}\right)^{\left(.3 + \frac{.7KL}{G}\right)}$$

$$= 1 - \left(\frac{1200}{1200+100}\right)^{\left[.3 + \frac{.7(10)(1200)}{100}\right]}$$

$$= 1 - (.92308)^{84.3} = .99883.$$

NOTE: THIS IS THE CONFIDENCE OF AT LEAST BREAKING EVEN IN THE LONG RUN.

THE LOGARITHMIC CONFIDENCE INTERPOLATION DIAGRAM FOR THIS EXAMPLE IS SHOWN IN FIGURE 2.

CONFIDENCE INTERPOLATION PAPER - FIRST QUADRANT
DESIGNED BY LEONARD G. JOHNSON