Statistical Bulletin
Reliability & Variation Research

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ENTROPY THEORY OF FATIGUE LIFE REDUCTION DUE TO INCLUSIONS

INTRODUCTION

Very often in the design of mechanisms or machines the design engineer is faced with the question of the durability of a metallic part or component which may not be completely clean materially. In such cases it is important to know how much fatigue life reduction to expect from such dirt elements as inclusions or impurities mixed in with the matrix metal. Knowing the relative density of impurities (i.e., what portion of the total volume they occupy) and the stress concentration factor due to such impurities, it is possible to predict from Entropy Theory just how much life is reduced from what it would be in a clean material.

The mathematics of such life reduction predictions is the subject of this bulletin.

GENERAL FORMULA

$$
\begin{aligned}
&(\text{RESULTANT ENTROPY TO X CYCLES}) \\
&\quad = (\text{FRACTION}_{\text{dirty}})(\text{DIRTY ENTROPY TO X CYCLES}) \\
&\quad + (\text{FRACTION}_{\text{clean}})(\text{CLEAN ENTROPY TO X CYCLES})
\end{aligned}
$$

ENTROPY THEORY OF FATIGUE LIFE REDUCTION DUE TO INCLUSIONS

In case the life at a fixed stress level has a two-parameter Weibull distribution of slope $\underline{b}$, and in case the S-N relation

is $$\text{LIFE} = \frac{\text{CONSTANT}}{S^m} \qquad (m > 0)$$

(S = Stress) , it follows that

$$(\text{Dirty Entropy to X Cycles}) = (\text{Clean Entropy to X Cycles})\lambda_{\text{dirty}}^{mb}$$

Where $$\lambda_{\text{dirty}} = \frac{S_{\text{dirty}}}{S_{\text{clean}}}$$

(Thus, λ_{dirty} is a Stress Concentration Factor at a Dirty Elemental Volume)

$$\mathcal{E} = \left(\frac{x}{\theta}\right)^b \quad \text{2-parameter Weibull Entropy}$$

Thus ,

$$\left(\frac{\text{Resultant Entropy to X Cycles}}{\text{Clean Entropy to X Cycles}}\right) = \text{Fraction}_{\text{clean}} + \text{Fraction}_{\text{dirty}}\lambda_{\text{dirty}}^{mb}$$

Therefore ,

$$\left(\frac{\text{Clean Characteristic Life}}{\text{Resultant Characteristic Life}}\right)^b = \text{Fraction}_{\text{clean}} + \text{Fraction}_{\text{dirty}}\ \lambda_{\text{dirty}}^{mb}$$

or

$$\text{Resultant Characteristic Life} = \frac{\text{Clean Characteristic Life}}{\left(\text{Fraction}_{\text{clean}} + \text{Fraction}_{\text{dirty}}\lambda_{\text{dirty}}^{mb}\right)^{\frac{1}{b}}}$$

EXAMPLE

A clean specimen has a Weibull slope of 1.3 and a Characteristic life of 1,000,000 cycles. What will be the Characteristic life of a specimen which has inclusions uniformly distributed in 1/1000 of its volume , if the Stress Concentration Factor due to such inclusions is $\lambda_{dirty} = 2.0$ and the S-N relation is

$$\text{Life} = \frac{\text{Constant}}{(\text{Stress})^{10}} \quad ?$$

Input Factors For the Example :

Clean Specimen :
- $b = 1.3$
- $\theta_{cl.} = 1{,}000{,}000$ cycles

Specimen with Inclusions:
- $\text{Fraction}_{dirty} = .001$
- Stress Concentration Factor at Inclusion $= \lambda_{dirty} = 2$

S - N Relation : $\text{Life} = \dfrac{\text{Constant}}{S^{10}}$

QUESTION : What is the Resultant Characteristic Life of the specimen with inclusions ?

SOLUTION TO EXAMPLE

Use the Formula

$$\theta_{Resultant} = \frac{\theta_{clean}}{\left[Fraction_{clean} + Fraction_{dirty}\ \lambda_{dirty}^{mb}\right]^{\frac{1}{b}}}$$

In this case :

$$\left\{ \begin{array}{l} \theta_{clean} = 1{,}000{,}000 \text{ cycles} \\ b = 1.3 \quad ; \quad m = 10 \\ Fraction_{dirty} = .001 \\ Fraction_{clean} = .999 \\ \lambda_{dirty} = 2 \end{array} \right\}$$

$$\therefore \quad \theta_{Resultant} = \frac{1{,}000{,}000}{\left[.999 + .001\ (2^{13})\right]^{1/1.3}}$$

$$= \frac{1{,}000{,}000}{(9.191)^{1/1.3}} = \underline{\underline{181{,}531 \text{ cycles}}} \quad \text{(ans.)}$$

ANOTHER EXAMPLE :

$$\left\{ \begin{array}{l} b = 2.5 \\ m = 6 \end{array} \right.$$

$\lambda = 1.5$

$\theta_{clean} = 10{,}000$ hrs.

$Fraction_{dirty} = .0005$

Predict : $\theta_{Resultant}$

SOLUTION :

$$\theta_{Resultant} = \frac{\theta_{clean}}{\left(.9995 + .0005(1.5)^{15}\right)^{1/2.5}}$$

$$= \frac{10{,}000}{1.08224} = \underline{\underline{9{,}240 \text{ hours}}} \quad \text{(ans.)}$$

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THE DC - 10 DIRGE
(FLIGHT 191 - IN MEMORIAM)

The engine roared and shook the wing.
The pylon bolt was crying and grim.
Help, help, I'm coming apart.
"With me like this, you must not start."
The pilot taxied ahead with his craft
as the bolt gave up with its sad last gasp.
The engine feeling free and wild ,
jumped up and over to the other side.
In a moment the runway felt the jolt ,
as the engine came down with the broken bolt.
The pilot swore at the lack of control ,
while a blasting inferno engulfed the whole.
Judgment is come, and we all must die ,
for nobody heard that bolt's last cry.
Oh foolish inspectors who habitually say ,
"Tomorrow we'll check , but not today."
Now you've found to your great shame ,
that no two bolts are ever the same.
Each bolt that's made comes from a lot
whose story is told by a WEIBULL PLOT.

Leonard G. Johnson
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