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# CONFIDENCE BAND CONSTRUCTION FOR "SUDDEN DEATH" POPULATION PLOTS ON PROBABILITY PAPER

## INTRODUCTION

The following question is often asked :

> "How can a confidence band be constructed for a "Sudden Death" population Weibull plot ?"

It is in answer to this particular question that this short bulletin has been prepared .

## "SUDDEN DEATH" POPULATION PLOT

(ORDER NUMBERS FOR DETERMINING MEDIAN RANKS)
( K = GROUP SIZE ) ( r = NO. OF FAILURES )

| FAILURE NO. | ORDER NUMBER IN $N = rK$ |
|---|---|
| 1 | $O_1 = 1$ |
| 2 | $O_2 = 1 + \frac{N}{1 + N - K}$ |
| 3 | $O_3 = 1 + \frac{N}{1 + N - K} + \frac{N(N - K)}{(1 + N - K)(1 + N - 2K)}$ |
| 4 | $O_4 = 1 + \frac{N}{1 + N - K} + \frac{N(N - K)}{(1+N-K)(1+N-2K)} + \frac{N(N - K)(N - 2K)}{(1+N-K)(1+N-2K)(1+N-3K)}$ |
| ETC . | ETC. |

Median Ranks of the failures are calculated by using a sample size $N = rK$, together with the order number $O_j$ for failure number $j$ .

EXAMPLE : (N = 40) (K = 8 = Group Size) (r = 5 Failures)

| FAILURE NO. | ORDER NO. IN 40 | MEDIAK RANK (BENARD'S FORMULA) |
|---|---|---|
| 1 | 1 | .0173 |
| 2 | $1 + \frac{40}{30} = 1 + 1.21212 = 2.21212$ | .0473 |
| 3 | $2.21212 + \frac{(40)(32)}{(33)(25)} = 3.76364$ | .0857 |
| 4 | $3.76364 + \frac{(40)(32)(24)}{(33)(25)(17)} = 5.95401$ | .1400 |
| 5 | $5.95401 + \frac{(40)(32)(24)(16)}{(33)(25)(17)(9)} = 9.84801$ | .2363 |

ORDER NUMBERS AND SAMPLE SIZES FOR THE CONFIDENCE BAND OF A "SUDDEN DEATH" POPULATION PLOT

| FAILURE NO. | SAMPLE SIZE | ORDER NO. FOR 5% AND 95% RANKS |
|---|---|---|
| 1 | N | 1 |
| 2 | N - K | $.3 + (O_2 - .3)\left(\frac{N - K + .4}{N + .4}\right)$ |
| 3 | N - 2K | $.3 + (O_3 - .3)\left(\frac{N - 2K + .4}{N + .4}\right)$ |
| 4 | N - 3K | $.3 + (O_4 - .3)\left(\frac{N - 3K + .4}{N + .4}\right)$ |
| . | . | . |
| . | . | . |
| . | . | . |
| r | N - (r - 1) K | $.3 + (O_r - .3)\left(\frac{N - (r - 1)K + .4}{N + .4}\right)$ |

To construct the 90% Confidence Band , use the order numbers in Column 3 together with the Sample Sizes in Column 2 to determine the 5% Rank and the 95 % Rank at each failure abscissa .

CONFIDENCE BAND DATA FOR THE EXAMPLE IN WHICH N = 40 IS BROKEN UP INTO 5 GROUPS OF 8 EACH , I. E., r = 5 AND K = 8

| FAILURE NO. | SAMPLE SIZE | ORDER NUMBER FOR 5% AND 95% RANKS |
|---|---|---|
| 1 | 40 | 1 |
| 2 | 32 | $.3 + (2.21212 - .3)(\frac{32.4}{40.4}) = 1.83348$ |
| 3 | 24 | $.3 + (3.76364 - .3)(\frac{24.4}{40.4}) = 2.39190$ |
| 4 | 16 | $.3 + (5.95401 - .3)(\frac{16.4}{40.4}) = 2.59519$ |
| 5 | 8 | $.3 + (9.84801 - .3)(\frac{8.4}{40.4}) = 2.28523$ |

Now to construct the 90 % Confidence Band for this population plot , we determine the

5% and 95% Ranks of #1 in 40 for Failure No. 1 ,

5% and 95% Ranks of #1.83348 in 32 for Failure No. 2 ,

5% and 95% Ranks of #2.39190 in 24 for Failure No. 3 ,

5% and 95% Ranks of #2.59519 in 16 for Failure No. 4 ,

5% and 95% Ranks of #2.28523 in 8 for Failure No. 5 ,

BY LINEAR INTERPOLATION IN 5% AND 95% RANK TABLES , WE FIND THE PROPER PLOTTING POSITIONS FOR THE FIVE FAILURES IN THE EXAMPLE WITH K = 8 AND r = 5 (N = 40) TO BE AS FOLLOWS :

| FAILURE NO. | 5 % RANK | 95 % RANK |
|---|---|---|
| 1 | .00128 | .07216 |
| 2 | .00993 | .13144 |
| 3 | .02282 | .20519 |
| 4 | .04082 | .31150 |
| 5 | .06485 | .50748 |

USING THESE CALCULATED 5% RANKS AND 95% RANKS , WE CONSTRUCT THE CORRECT CONFIDENCE BAND FOR A "SUDDEN DEATH" POPULATION LINE FOR ANY CASE IN WHICH K = 8 , r = 5 , AND N = 40 .