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MISCELLANEOUS ITEMS REGARDING RENEWAL THEORY

ITEM # 1

NON-DEBITED WEIBULL RENEWAL

FOR THE FIRST θ HOURS THE PART WHICH IS NON-DEBITED IN A MACHINE SHOWS

$$\left(\frac{x}{\theta}\right)^{b} \quad \text{FAILURE PER MACHINE.}$$

$$\left(\begin{array}{lll} b & = & \text{WEIBULL SLOPE OF THE PART} \\ \theta & = & \text{CHARACTERISTIC LIFE OF THE PART} \\ & & \quad (\text{MINIMUM LIFE} = 0) \end{array}\right)$$

BEYOND THE FIRST θ HOURS, THE FAILURES PER MACHINE (OF THE PART) BECOMES

$$\left(\frac{x}{\theta}\right).$$

IF $b > 1$ (I. E., IF THE PART HAS A WEAR-OUT WEIBULL DISTRIBUTION), THEN IN ELAPSED TIME x THE FAILURES PER MACHINE (OF THE PART) CAN BE WRITTEN AS

$$\text{FAILURES/MACHINE} = \left(\frac{x}{\theta}\right)^{\gamma}$$

FAILURES/MACHINE =

WITH $\gamma = 1 + (b-1)\, e^{-4(b-1)(x/\theta)^{K}}$

WHERE $K = 2 + \dfrac{\ln (b-1)}{\ln 2} - \dfrac{\ln \ln (b-1/.99b-1)}{\ln 2}$

(FOR $1 \leq b \leq 1.010101$ THE VALUE OF γ IS UNITY.)

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ITEM # 2

FAILURES PER

FAILURES PER MACHINE OF NON-DEBITED PART WITH $b < 1$ AND CHARACTERISTIC LIFE θ .

IN ELAPSED MACHINE TIME x :

$$\text{FAILURES/MACHINE} = \left(\frac{x}{\theta}\right)^{\gamma}$$

WITH

$$\gamma = 1 + (b - 1)\, e^{-4(1-b)\left(\frac{x}{\theta}\right)^{K}}$$

WHERE

$$K = 2 + \frac{\ln(1 - b)}{\ln 2} - \frac{\ln \ln\left(\frac{1 - b}{1 - 1.01\, b}\right)}{\ln 2}$$

NOTE : FOR $.99 \leq b \leq 1.00$, THE VALUE OF γ IS UNITY .

$$\left(\begin{array}{l} b = \text{WEIBULL SLOPE OF PART} \quad < 1 \\ \theta = \text{CHARACTERISTIC LIFE OF PART} \\ \quad\quad (\text{MINIMUM LIFE} = 0) \end{array}\right)$$

ITEM # 3

FULLY DEBITED WEIBULL RENEWAL

A FULLY DEBITED PART IN A MACHINE ALWAYS SHOWS $\left(\frac{x}{\theta}\right)^{b}$ FAILURES PER MACHINE (ON THE AVERAGE) IN ELAPSED MACHINE SERVICE TIME x , IF THE PART HAS A WEIBULL SLOPE $\underline{b}$ AND A CHARACTERISTIC LIFE $\underline{\theta}$. THE MINIMUM LIFE IS ASSUMED TO BE ZERO.

ITEM # 4

LET A SYSTEM HAVE A TWO-SLOPE MODEL ON WEIBULL PAPER WHICH STARTS WITH

$$F_1(x) = 1 - e^{-\left(\frac{x}{\theta_1}\right)} \quad \left(\begin{array}{l}\text{SLOPE UNITY ;}\\ \text{(A DEBUGGED SYSTEM)}\end{array}\right)$$

SUPPOSE THE TWO-SLOPE MODEL ENDS WITH THE WEIBULL FUNCTION

$$F_2(x) = 1 - e^{-\left(\frac{x}{\theta_2}\right)^b} \quad \left(\begin{array}{l}\text{SLOPE } b > 1\\ \text{(WEAR-OUT STAGE)}\end{array}\right)$$

THEN, THE INTERSECTION POINT OF THE TWO WEIBULL LINES WILL BE AT ABSCISSA

$$x_1 = \theta_1 \rho^{\left(\frac{b}{b-1}\right)}$$

WHERE ρ = CHARACTERISTIC LIFE RATIO $\left(\frac{\theta_2}{\theta_1}\right)$.

FURTHERMORE, THE ORDINATE OF THE INTERSECTION POINT IS AT QUANTILE

$$Q_1 = 1 - e^{-\rho\left(\frac{b}{b-1}\right)}$$

IF FAILURES IN THE SYSTEM ARE <u>FULLY DEBITED</u>, THEN FOR THE FIRST x_1 HOURS, THE FAILURES PER MACHINE WILL BE (x/θ_1) ; $(0 \leq x \leq x_1)$

AFTER x_1 HOURS, WE WILL FIND THAT

$$\text{FAILURES PER MACHINE} = (x/\theta_2)^b \quad ; \quad (x > x_1)$$

ITEM # 5

GENERAL FULLY DEBITED RENEWAL THEORY

<u>THEOREM:</u> WHATEVER CDF $F(x)$ A FAILURE MODE HAS, IF ALL SUCH FAILURES OF THIS MODE WHEN REPAIRED BECOME FULLY DEBITED, THE FAILURES PER MACHINE IS GIVEN BY

$$\ln\left(\frac{1}{1 - F(x)}\right) \text{ FOR ALL } x.$$

GENERAL NON-DEBITED RENEWAL THEORY

(FOR MINIMUM LIFE=0)

THEOREM : IF A MACHINE HAS A SUB-SYSTEM IN IT WHICH HAS A CDF F(x) , AND IF THE SUB-SYSTEM IS FULLY NON-DEBITED UPON BEING REPLACED DUE TO FAILURE , IT FOLLOWS THAT THE FAILURES PER MACHINE DUE TO THE SUB-SYSTEM IS

$$\ln\left(\frac{1}{1 - F(x)}\right) \quad \text{UP TO 1 FAILURE PER MACHINE,}$$

I. E., UP TO $x_1 = \mathcal{E}^{-1}(1)$, WHERE $\mathcal{E}^{-1}(x)$ IS THE INVERSE ENTROPY FUNCTION ,

AFTER 1 FAILURE PER MACHINE , THE FORMULA FOR FAILURES PER MACHINE IS

$$\text{FAILURES/MACHINE} = (x/x_1) \text{ , WHERE}$$

$$x_1 = B_{63.2\%} \text{ LIFE OF THE SUB-SYSTEM}$$

AND $x > x_1$ IN THE FORMULA (x/x_1) .