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FAILURES PER MACHINE AS PREDICTED FROM FULLY NON-DEBITTED RENEWAL THEORY

INTRODUCTION

When a new part takes the place of a failed part in a machine system, it has been found in studying repeated failures of that same type of part in the same location within the machine, that the part can be assumed to start afresh without penalty (DEBIT) against it regardless of the age of the rest of the machine. In other words, a newly installed part in a used machine usually behaves in the same fashion as if it had been installed into a new machine. Simply stated, the age of the machine should not be debited to the newly installed part, but each newly installed part can be assumed to go back to age zero as far as the part itself is concerned.

THE BEHAVIOUR OF THE HAZARD PLOT OF A NON-DEBITED PART WHICH EXPERIENCES PERIODIC FAILURES

The first time a specific part is installed into a new machine, the hazard curve will show an increasing hazard curve will show an increasing hazard in case of wear-out, such as in Figure 1 :

STAGE 1 (ORIGINAL PART)

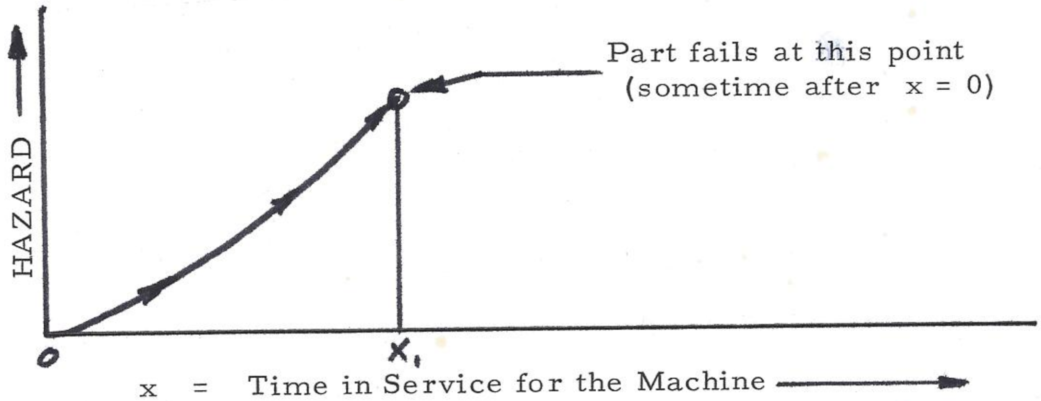

FIGURE 1

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The first time the part fails will be at some time of service x_1 after $x = 0$. Then the part is replaced, and the hazard of the replacement part drops down to zero again, as shown in Figure 2. From zero hazard the hazard rises for the replacement part at the same rate of wear-out as it did for the initial installation (see Figure 2).

STAGES 1 & 2 (FIRST TWO PARTS)

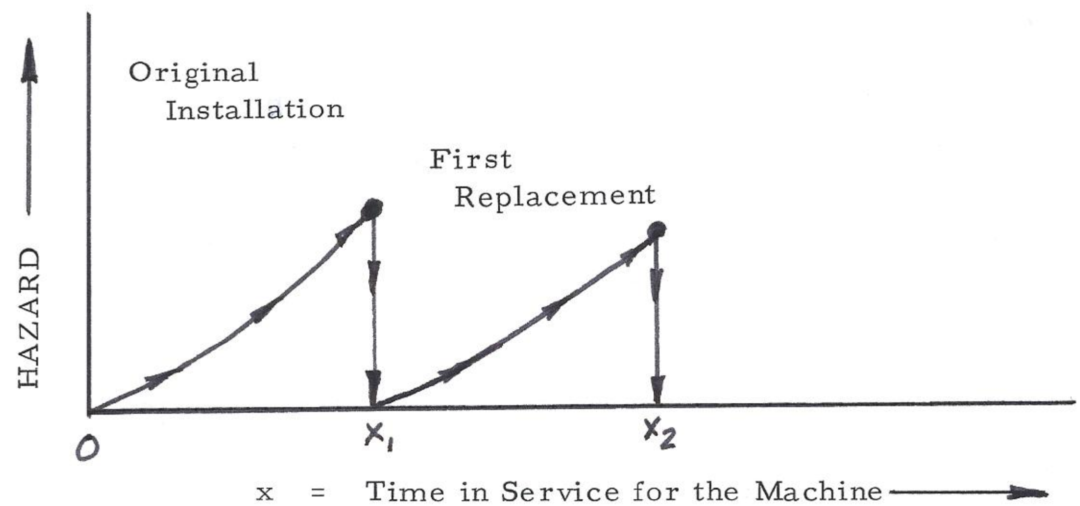

FIGURE 2

The first replacement part fails at x_2 hours of machine service , or $(x_2 - x_1)$ hours of service on itself. At x_2 hours on the machine a second replacement is made for the failed part , and the hazard drops down to zero again , and then starts rising at the same rate as it did earlier with the first two installations (see Figure 3) .

STAGES 1 , 2 , AND 3 (FIRST THREE PARTS)

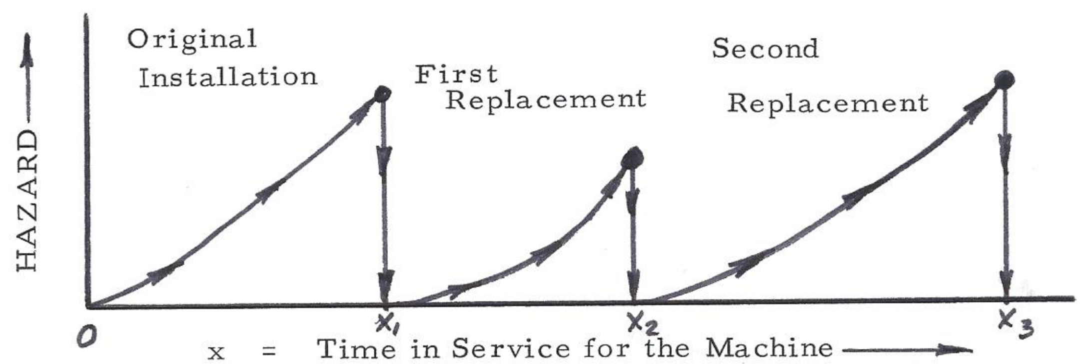

FIGURE 3

At some time x_3 of machine service the third part to be installed in the machine will fail and be replaced with a fourth part which starts again at zero hazard and shows the same rate of hazard growth as the first three parts did (see Figure 4).

STAGES 1 , 2, 3, AND 4, (FIRST FOUR PARTS)

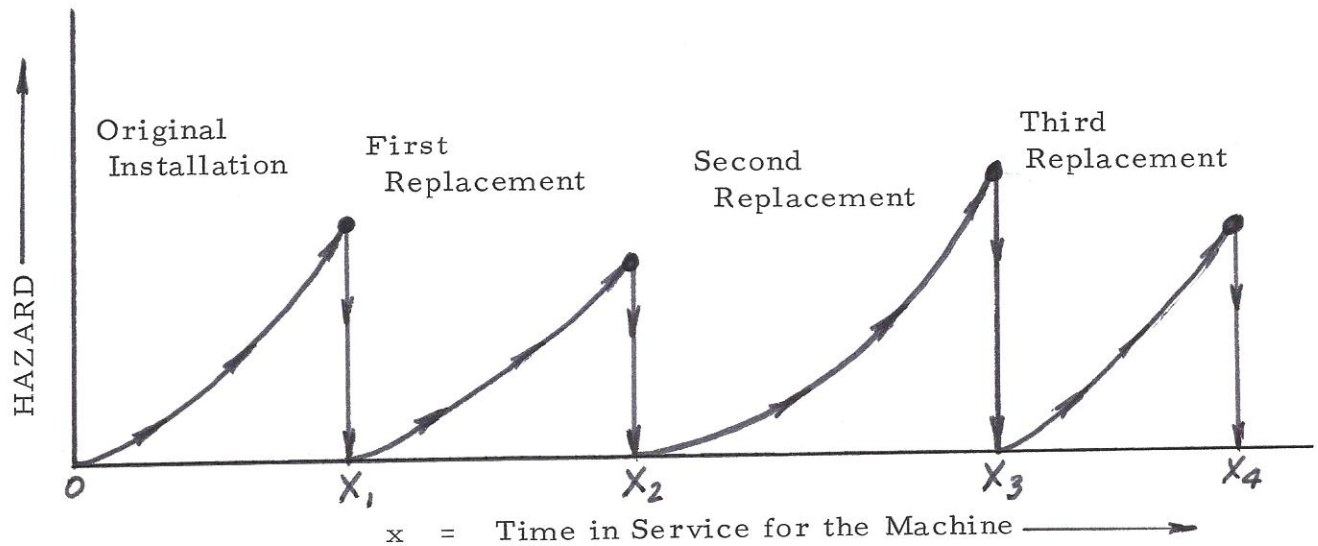

FIGURE 4

THE RESULTING AVERAGE HAZARD AFTER MANY STAGES OF BEHAVIOUR SUCH AS DEPICTED IN FIGURE 1 THROUGH 4 AVERAGED OVER MANY SIMILAR MACHINES

After many stages of behaviour as depicted in Figure 1 through 4 , it follows that the RESULTANT AVERAGE HAZARD CURVE for successive replacements of the same non-debited part will average out to a smooth hazard curve of the type shown in Figure 5.

(It is assumed that the part has a Weibull Slope $b > 1$ (for wear-out) , with a Characteristic Life equal to θ .)

AVERAGE RESULTANT HAZARD FOR NON-DEBITED WEAR-OUT PARTS

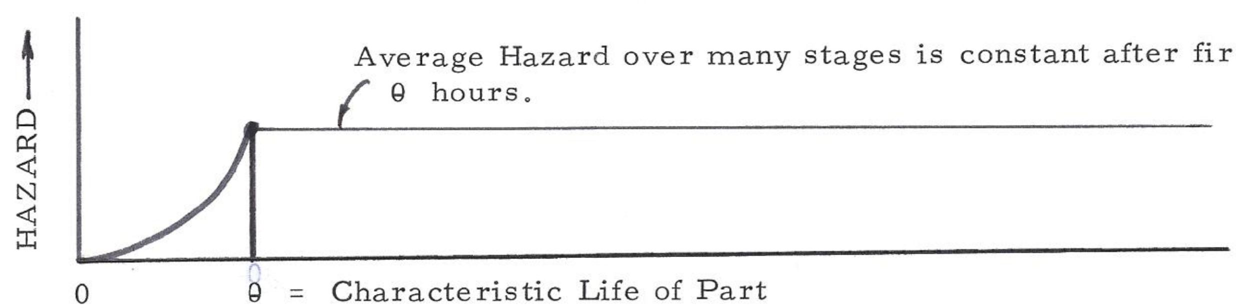

FIGURE 5

RESULTANT AVERAGE FAILURES PER MACHINE

A formula for the RESULTANT AVERAGE HAZARD CURVE for a Non-debited part must show that initially the part produces failures per machine according to the ENTROPY for Weibull Parameters (b, θ) , i. e., initially (for the first θ hours),

$$\text{FAILURES/MACHINE} = \left(\frac{x}{\theta}\right)^b$$

After $x = \theta$ hours , the Hazard levels to a constant value (on the average) , when averaged over many similar machines with this same type of part in the same location in each machine. This average will be such that for $x > \theta$, the failures per machine will average out to (x/θ) , i. e., after θ hours , we can write

$$\text{FAILURE/MACHINE} = \left(\frac{x}{\theta}\right) .$$

This indicates that a Non-debited part eventually will always exhibit a log-log plot of slope UNITY for failures per machine after the first θ hours of machine service .

A GOOD EMPIRICAL FORMULA FOR FAILURES PER MACHINE

We can now write a mathematical expression for failures per machine due to a Non-debited part which is repeatedly replaced. This formula states that

$$\text{FAILURES/MACHINE} = \left(\frac{x}{\theta}\right)^{\gamma} ,$$

where γ is initially (at $x = 0$) equal to the Weibull Slope b of the part , and thereafter γ decays down to a value of unity at $x = \theta$, and remains at unity for any x beyond θ . We can write a reasonable formula for γ as follows :

(This is in case the part has a Weibull Slope $b > 1$.)

$$\gamma = 1 + (b-1)e^{-4(b-1)\left(\frac{x}{\theta}\right)^K}$$

Where $$K = 2 + \frac{\ln(b-1)}{\ln 2} - \frac{\ln \ln\left(\frac{b-1}{.99b-1}\right)}{\ln 2}$$

NOTE : for $b = 1$, $\gamma = 1$ (always)

A b value from 1 to 1.010101 is considered to give $\gamma = 1$ (always).

APPLYING THE FORMULA TO A SPECIAL CASE

Let us take the case in which the Non-debited part has a wear-out Weibull Distribution with slope $b = 3$.

Then , according to the formula , we obtain the following tabulation of failures per machine with increasing machine service time x :

x	γ	$\left(\frac{x}{\theta}\right)^{\gamma}$ = FAILURES / MACHINE (part has b=3)
0	3.0	0
.25θ	2.99994	0.0156
.50θ	2.97000	0.1276
.75θ	2.10595	0.5456
1.00θ	1.00067	1.0000
1.25θ	1.00	1.25
1.50θ	1.00	1.50
1.75θ	1.00	1.75
2.00θ	1.00	2.00
2.25θ	1.00	2.25
2.50θ	1.00	2.50
2.75θ	1.00	2.75
3.00θ	1.00	3.00
3.25θ	1.00	3.25
3.50θ	1.00	3.50
3.75θ	1.00	3.75
4.00θ	1.00	4.00

From this table we obtain Figure 6 for the log-log plot of Failures/Machine versus the machine service time x .

FAILURES PER MACHINE DUE TO A NON-DEBITED PART WITH A WEIBULL SLOPE b = 3 AND A CHARACTERISTIC LIFE OF θ .

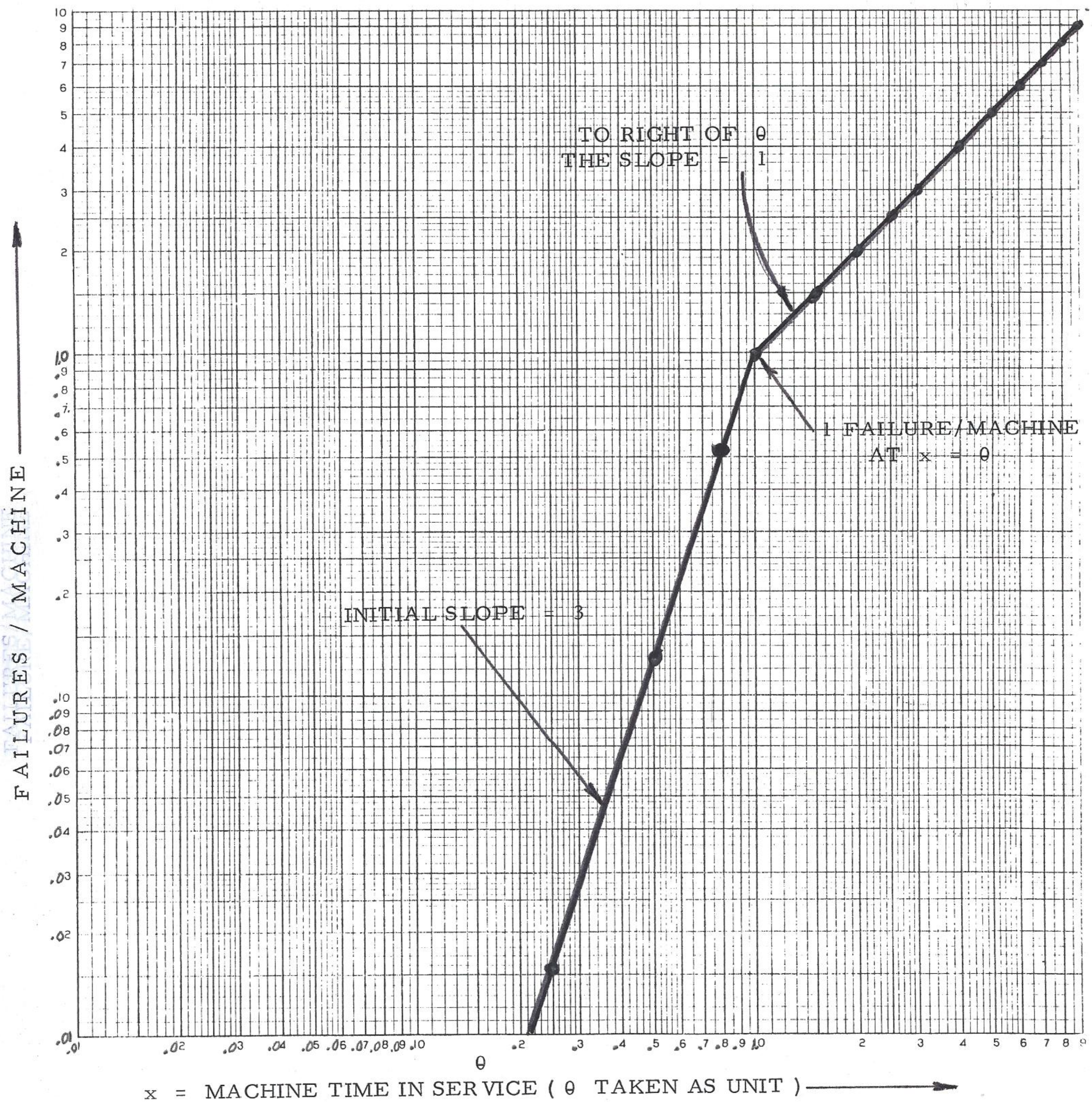

FIGURE 6

APPENDIX

FAILURES PER MACHINE ON A NON-DEBITED PART WITH WEIBULL SLOPE $b < 1$ AND A CHARACTERISTIC LIFE θ

$(b < 1)$

$$\text{FAILURES/MACHINE (IN ELAPSED TIME } x\text{)} = \left(\frac{x}{\theta}\right)^{\gamma}$$

WHERE

$$\gamma = 1 + (b-1)\, e^{-4(1-b)\left(\frac{x}{\theta}\right)^{K}}$$

WITH

$$K = 2 + \frac{\ln(1-b)}{\ln 2} - \frac{\ln\ln\left(\frac{1-b}{1-1.01b}\right)}{\ln 2}$$

NOTE: According to the above, it is assumed that for $.99 < b < 1.00$ the value of $\gamma = 1$ (always).