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WEIBULL ANALYSIS VIA THE SR-51 - II CALCULATOR

INTRODUCTION

Weibull Analysis is conventionally done graphically by means of straight lines fitted to life test data or field failure data on Weibull Probability paper. When looking at data plotted on Weibull Probability paper with the naked eye, it is quite an arbitrary process , which differs from person to person, when we decide the direction and position of the line which is intended to fit the data. The subjective nature of this naked eye curve fitting makes it quite undesirable and often times questionable, especially in cases where the data possesses so much scatter that no two individuals could agree on the best fitting line by looks alone .

As a more scientific alternative to naked eye curve fitting , we propose the method of LEAST SQUARES REGRESSION ANALYSIS in fitting a straight line to data plotted on Weibull Probability paper. This can be done with any calculator has a built-in linear regression program.

USING CALCULATORS WITH BUILT-IN LINEAR REGRESSION ANALYSIS

As stated in the introduction , Weibull Lines can be fitted with appropriate parameters by using any calculator with a built-in linear regression program. The only modifications needed are as follows :

(1) Before entering an abscissa X_i , which represents a time to failure , first take its natural logarithm $\ln X_i$. Then enter this natural logarithm as the abscissa for linear regression analysis.

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(2) Before entering an ordinate (MEDIAN RANK) , calculate the value of $\ln \ln \frac{1}{1 - \text{Median Rank}}$, and then enter this quantity as the Ordinate for the linear regression analysis. Each value X_i will have its own $(\text{Median Rank})_i$.

After modifying all possible pairs (X_i , Median Rank_i) by first determining $\ln X_i$ and $\ln \ln \frac{1}{1 - \text{Median Rank}_i}$ and entering these latter quantities as absicissas and ordinates for linear regression analysis , we find the Weibull Slope Parameter by simply taking the Slope of the regression line. Furthermore , we determine the Intercept of the regression line , and then it follows that the Characteristic Life θ is given by the formula

$$\theta = e^{-\frac{\text{intercept}}{\text{slope}}}$$

Thus , b = slope of the regression line

and θ = exp(-intercept/slope)

AN ACTUAL EXAMPLE WITH THE SR-51-II ELECTRONIC CALCULATOR

Suppose we have the following sample of five items which have been tested to failure , together with their Median Ranks from a Median Rank Table :

FAILURE NUMBER	HOURS TO FAILURE	MEDIAN RANK
1	51	.1294
2	97	.3138
3	150	.5000
4	220	.6862
5	300	.8706

STEPS IN THE WEIBULL ANALYSIS EXAMPLE VIA THE SR-51-II

ENTERING THE DATA

	Keys	DISPLAY WINDOW READS
press	[5] [1] [ln x] [x ⇌ y]	0.
press	[•] [8] [7] [0] [6] [1/x] [ln x] [ln x] [Σ+] This is (1 - .1294) (subtraction must be done mentally)	1.
press	[9] [7] [ln x] [x ⇌ y]	4.931825633
press	[•] [6] [8] [6] [2] [1/x] [ln x] [ln x] [Σ+] This is (1-.3138) (subtraction must be done mentally)	2.
press	[1] [5] [0] [ln x] [x ⇌ y]	5.574710979
press	[•] [5] [1/x] [ln x] [ln x] [Σ+] This is (1-.5) (subtraction must be done mentally)	3.
press	[2] [2] [0] [ln x] [x ⇌ y]	6.010635294
press	[•] [3] [1] [3] [8] [1/x] [ln x] [ln x] [Σ+] This is (1-.6862) (subtraction must be done mentally)	4.
press	[3] [0] [0] [ln x] [x ⇌ y]	6.393627546
press	[•] [1] [2] [9] [4] [1/x] [ln x] [ln x] [Σ+] This is (1-.8706) (subtraction must be done mentally)	5.

This concludes the entry of the 5 pairs (X_i , Median $Rank_i$) .

READING THE WEIBULL PARAMETERS FROM THE CALCULATOR

TO FIND THE WEIBULL SLOPE b :

			DISPLAY WINDOW READS
press	2nd	slope ÷	1.490170543

This is the Weibull Slope

Thus , the Weibull Slope = b = 1.490170543

TO FIND THE CHARACTERISTIC LIFE θ :

					DISPLAY WINDOW READS
press	0	2nd	x' −	e^x	191.0669461

This is the characteristic life

Thus , Characteristic Life = θ = 191.0669461 hours

This completes the determination of Weibull Parameters fitting the data sample (51, 97, 150, 220, 300) hours. Any other data sample can be analyzed the same way, as long as each life value in the sample is given the appropriate Median Rank (even when there are Suspended Items). In case there is a positive Minimum Life (3-parameter Weibull function) , the Minimum Life must be subtracted from the given life values before proceeding with the regression analysis .

READING THE GOODNESS OF FIT (CORRELATION COEFFICIENT) FROM THE CALCULATOR

To find the GOODNESS OF FIT for the line fitted to the data , do the following :

			DISPLAY WINDOW READS
press	2nd	corr y^x	.9991901964

Thus , GOODNESS OF FIT = Correlation Coefficient
= .9991901964

NOTE : We consider a Correlation Coefficient of at least $\left(1 - \frac{1}{2N}\right)$ for N Plotted Points to be a satisfactory Goodness of Fit . In the example of 5 items , the correlation coefficient should be at least $1 - 1/10 = .9$. Since .9991901964 $\geq$ .9 , we conclude the fit is satisfactory .

DISPLAY WINDOW READS

TO FIND THE B_{10} LIFE

press [.] [9] [1/x] [ln x] [ln x] [2nd] [x'] [e^x] — 42.20265255

This is (1 - .1)

Thus , B_{10} = 42.20265255 hours

TO FIND THE MEDIAN LIFE

press [.] [5] [1/x] [ln x] [ln x] [2nd] [x'] [e^x] — 149.4064128

This is (1 - .5)

Thus , Median Life = B_{50} = 149.4064128 hours

TO FIND THE B_{90} LIFE

press [.] [1] [1/x] [ln x] [ln x] [2nd] [x'] [e^x] — 334.3917226

This is (1 - .9)

Thus , B_{90} = 334.3917226 hours

APPENDIX

(THE MATHEMATICAL BASIS OF WEIBULL REGRESSION)

The Two-Parameter Weibull Cumulative Distribution Function is

$$F(x) = 1 - e^{-\left(\frac{x}{\Theta}\right)^{b}} \qquad (1)$$

Where x = Time to Failure (in a life test)

b = Weibull Slope

Θ = Characteristic Life

By transposition in (1) :

$$1 - F = e^{-\left(\frac{x}{\Theta}\right)^{b}}$$

$$\frac{1}{1 - F} = e^{+\left(\frac{x}{\Theta}\right)^{b}}$$

Taking the Natural Logarithm of both sides :

$$\ln \frac{1}{1 - F} = \left(\frac{x}{\Theta}\right)^{b}$$

Taking the Natural Logarithm of both sides again :

$$\ln \ln \frac{1}{1 - F} = b \ln x - b \ln \Theta$$

slope = b intercept = $- b \ln \Theta$

$$\ln \Theta = - \frac{\text{intercept}}{b} = - \frac{\text{intercept}}{\text{slope}}$$

$$\therefore \Theta = e^{-\frac{\text{intercept}}{\text{slope}}}$$