Statistical Bulletin
Reliability & Variation Research

DETROIT RESEARCH INSTITUTE
21900 GREENFIELD ROAD • OAK PARK, MICHIGAN 48237 • (313) 968-1818

LEONARD G. JOHNSON
EDITOR

WANG H. YEE
DIRECTOR

Vol. 7
Bulletin 2

May, 1977
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USE OF THE TABLE OF "EXACTLY" COEFFICIENTS IN SYSTEMS RELIABILITY PROBLEMS

When we have a collection (system) of several items, and each of these items has its own specific reliability (probability) of surviving a specific number of hours, we can ask the following questions :

(1) What is the probability that EXACTLY NONE of the items will survive the required hours of operation ?

(2) What is the probability that EXACTLY ONE of the items will survive the required hours of operation ?

(3) What is the probability that EXACTLY TWO of the items will survive the required hours of operation ?

(4) What is the probability that EXACTLY THREE of the items will survive the required hours of operation ?

(5) What is the probability that EXACTLY FOUR of the items will survive the required hours of operation ?

etc. etc. etc.

A NUMERICAL PROBLEM EXAMPLE

A collection of four items is designed into a certain assembly. The four individual items have the reliabilities for surviving the required time in service are on the following page :

Published 8 times a year by the Detroit Research Institute. Subscription Rate: One year $40.00. Quantity rates on request.
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ITEM NO.	RELIABILITY TO REQUIRED SURVICE TIME
1	$R_1 = .70$
2	$R_2 = .80$
3	$R_3 = .75$
4	$R_4 = .90$

By SUBTRACTING each of these Reliabilities from unity , we can form the following table of FAILURE PROBABILITIES within the required service time :

ITEM NO.	PROBABILITY OF FAILURE TO LAST REQUIRED SERVICE TIME
1	$F_1 = .30$
2	$F_2 = .20$
3	$F_3 = .25$
4	$F_4 = .10$

There are 16 possible outcomes. This follows from the fact if each of N items can have two possible outcomes each (i. e., survive or fail), then there are 2^N possible outcomes.

For N = 4, this becomes $2^4 = 16$ outcomes. These 16 outcomes are symbolically listed on the next page :

TABLE I

OUTCOME NO.	PROBABILITY OF OUTCOME
1	$F_1F_2F_3F_4$ = .0015
2	$R_1F_2F_3F_4$ = .0035
3	$R_2F_1F_3F_4$ = .0060
4	$R_3F_1F_2F_4$ = .0045
5	$R_4F_1F_2F_3$ = .0135
6	$R_1R_2F_3F_4$ = .0140
7	$R_1R_3F_2F_4$ = .0105
8	$R_1R_4F_2F_3$ = .0315
9	$R_2R_3F_1F_4$ = .0180
10	$R_2R_4F_1F_3$ = .0540
11	$R_3R_4F_1F_2$ = .0405
12	$F_1R_2R_3R_4$ = .1620
13	$F_2R_1R_3R_4$ = .0945
14	$F_3R_1R_2R_4$ = .1260
15	$F_4R_1R_2R_3$ = .0420
16	$R_1R_2R_3R_4$ = .3780
	TOTAL = 1.0000

From Table I we can easily see that

Prob. (Exactly NONE Survive) = Probability of Outcome No. 1
= .0015

Prob. (Exactly ONE Survives) = Sum of the probabilities of outcomes 2 through 5 = .0275

Prob. (Exactly TWO Survive) = Sum of the probabilities of Outcomes 6 through 11 = .1685

Prob. (Exactly THREE Survive) = Sum of the probabilities of Outcomes 12 through 15 = .4245

Prob. (Exactly FOUR Survive) = Probability of Outcome 16
= .3780

<u>NOTE:</u> The sum of these five "EXACTLY" probabilities is UNITY.

These same results could have been obtained by using the the Table of "EXACTLY" Coefficients on the next page :

TABLE II

TABLE OF "EXACTLY" COEFFICIENTS

	constant	$S^{(1)}$	$S^{(2)}$	$S^{(3)}$	$S^{(4)}$	$S^{(5)}$	$S^{(6)}$	$S^{(7)}$	$S^{(8)}$	$S^{(9)}$	$S^{(10)}$
Exactly None	+1	-1	+1	-1	+1	-1	+1	-1	+1	-1	+1
Exactly One		+1	-2	+3	-4	+5	-6	+7	-8	+9	-10
Exactly Two			+1	-3	+6	-10	+15	-21	+28	-36	+45
Exactly Three				+1	-4	+10	-20	+35	-56	+84	-120
Exactly Four					+1	-5	+15	-35	+70	-126	+210
Exactly Five						+1	-6	+21	-56	+126	-252
Exactly Six							+1	-7	+28	-84	+210
Exactly Seven								+1	-8	+36	-120
Exactly Eight									+1	-9	+45
Exactly Nine										+1	-10
Exactly Ten											+1

$S^{(1)}$ = Sum of R's = $R_1 + R_2 + R_3 + R_4 = 3.15$

$S^{(2)}$ = Sum of All Pairs of R's = $R_1R_2 + R_1R_3 + R_1R_4 + R_2R_3 + R_2R_4 + R_3R_4 = 3.71$

$S^{(3)}$ = Sum of All Triplets of R's = $R_1R_2R_3 + R_1R_2R_4 + R_1R_3R_4 + R_2R_3R_4$
$= 1.9365$

$S^{(4)}$ = Sum of All Quadruplets of R's = $R_1R_2R_3R_4 = .3780$

According to Table II :

Prob. (Exactly None Survive) $= 1 - S^{(1)} + S^{(2)} - S^{(3)} + S^{(4)}$
$= 1 - 3.15 + 3.71 - 1.9365 + .3780$
$= .0015$

Prob. (Exactly One Survives) $= S^{(1)} - 2S^{(2)} + 3S^{(3)} - 4S^{(4)}$
$= 3.15 - 2(3.71) + 3(1.9365) - 4(.3780)$
$= .0275$

Prob. (Exactly Two Survive) $= S^{(2)} - 3S^{(3)} + 6S^{(4)}$
$= 3.71 - 3(1.9365) + 6(.3780)$
$= .1685$

Prob. (Exactly Three Survive) $= S^{(3)} - 4S^{(4)} = 1.9365 - 4(.3780)$
$= .4245$

Prob. (Exactly Four Survive) $= S^{(4)} = .3780$

These are the same results we obtained earlier. Table II is extended by forming Binomial Coefficients for the i^{th} power of a Binomial in the column headed by $S^{(i)}$ and alternating the signs (starting with +1 at the top if i is even, and starting with -1 at the top if i is odd).

The diagonal always has +1's in it.

For N items we stop with $S^{(N)}$.

Only the EXACTLY NONE case uses the initial constant term.