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Reliability & Variation Research

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COMPLIANCE RELIABILITY
QUESTIONS AND CRITERIA

TYPES OF COMPLIANCE QUESTIONS

TYPE 1

GIVEN THE RELIABILITY DESIRED TO THE STANDARD WHAT CONFIDENCE FOR THIS DESIRED RELIABILITY DOES A SET OF TEST DATA YIELD ?

TYPE 2

GIVEN THE CONFIDENCE DESIRED FOR A RELIABILITY STATEMENT TO THE STANDARD. WHAT RELIABILITY LEVEL WILL HAVE THIS DESIRED CONFIDENCE ACCORDING TO THE TEST DATA ?

TYPE 3

GIVEN THE RELIABILITY DESIRED TO THE STANDARD WHAT EVIDENCE OF THIS DESIRED RELIABILITY IS PROVIDED BY THE TEST DATA ?

TYPE 4

GIVEN THE DESIRED AMOUNT OF EVIDENCE WE WANT FOR A RELIABILITY STATEMENT TO THE STANDARD.
WHAT RELIABILITY LEVEL WILL HAVE THIS DESIRED AMOUNT OF EVIDENCE ACCORDING TO THE TEST DATA ?

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October , 1976
Page 2

ANOTHER EXAMPLE OF LOWER LIMIT ASSURANCE TESTING

SAMPLE SIZE = 9

1500 hrs.
3101 hrs.
4752 hrs.
6206 hrs.
8103 hrs. (X_o = Standard = 500 hrs.)
10,400 hrs.
12,850 hrs.
16,020 hrs.
21,509 hrs.

Compliance will net a $28 million profit ; Non-compliance will cause a $7.5 million loss .

Do the above data provide sufficient evidence to assure at least 10 to 1 odds in favor of monetary gain from the sale of fifty of these items ?

SOLUTION : $E_{req.} = \ln\left(10 \times \frac{7,500,000}{28,000,000}\right) = \underline{.98528}$

$\rho_1 = \left(\frac{X_1}{X_o}\right) = 1500/500 = 3$; Weibull Slope = 1.32

R = Required Reliability = 50/51 = .98039 ; N = 9

$$E_{accum.(log.par.)} = 1.8138\sqrt{N}\left\{\ln\ln\frac{1}{R} + b\ln\rho_1 - \ln\ln\left(\frac{N+.4}{N-.3}\right)\right\}$$

$$= 1.8138\sqrt{9}\left[\ln\ln\frac{1}{.98039} + 1.32\ln 3 - \ln\ln\frac{9.4}{8.7}\right]$$

$$= \underline{.47492}$$

CONCLUSION : Since $E_{accum.} < E_{req.}$ we conclude that there is still insufficient evidence of compliance to the required 500 hrs. of life.

[Only 62% Conf. (actual) when 73% Conf. is required .]

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Volume 6
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October , 1976
Page 3

AN UPPER LIMIT COMPLIANCE PROBLEM

The deceleration of an occupant's forehead against a motor vehicle's dash padding must not exceed 80 g's in a 30 mph barrier crash test.

8 vehicles with dummies were given such a barrier crash test. The measured decelerations were (in numerical order) :

53.2 g's
56.0 g's
57.5 g's
59.5 g's
61.3 g's
62.1 g's
63.4 g's
64.9 g's

The desired reliability of not exceeding 80 g's is .999999. Find the log parametric confidence that this desired reliability will be realized.

SOLUTION:

	Vehicle		Median Rank	
	53.2		.083	
	56.0		.202	
	57.5		.321	
	59.5		.440	
The data	61.3	plotted at median	.560	ranks yields
	62.1		.679	
	63.4		.798	
	64.9		.917	

Weibull Parameters $\left\{ \begin{array}{l} b = 16.23 \\ \theta = 61.5467 \end{array} \right\}$. (intcp. = -66.8643)

$N = 8$; $R = .999999$ (desired) ; $b = 16.23$; $P_N = \left(\frac{X_8}{X_o}\right) = \frac{64.9}{80} = .81125$

Hence, the Log Parametric Evidence is

$$E = 1.8138\sqrt{8}\left\{\ln\ln\frac{8.4}{.7} - \ln\ln\frac{1}{1 - .999999} - 16.23\ln(.81125)\right\}$$

$$= \underline{8.61575}.$$

Therefore , the Log Parametric Confidence is

$$C = \frac{1}{1 + e^{-E}} = \underline{.9998} \text{ (ans.)}$$

NOTE: The Non-Parametric Confidence is .9996.

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CALCULATING THE ACCUMULATED EVIDENCE FROM A DATA SET IN TERMS OF THE MEAN, STANDARD DEVIATION, & SKEWNESS

LOWER LIMIT TESTING

Let N = Sample Size ; $\bar{X}$ = Sample Mean

S = Sample Standard Deviation ; α_3 = Sample Skewness

${}_{\alpha_3}t_{1-R}$ = t-Score to $(1 - R)$ level in a distribution of skewness α_3

Then, the Evidence in favor of R (Standard) is

$$E = 1.8138\sqrt{N}\left[{}_{\alpha_3}t_{1-R} + \left(\frac{\bar{X} - \text{STD.}}{S}\right)\right]$$

UPPER LIMIT TESTING

The formula for Evidence in favor of R(STD.) is

$$E = 1.8138\sqrt{N}\left[\left(\frac{\text{STD.} - \bar{X}}{S}\right) - {}_{\alpha_3}t_{R}\right]$$

where ${}_{\alpha_3}t_{R}$ = t-Score to R Level in a distribution of skewness α_3.

(R = Desired Reliability)

LOWER LIMIT COMPLIANCE EXAMPLE USING MEAN, STANDARD DEV., AND SKEWNESS

PROBLEM

Suppose a guaranteed minimum life is 1200 hours (STD.), and suppose we run 6 items and find the following hrs. to failure :

1970 hrs., 2350 hrs., 2700 hrs., 3040 hrs., 3210 hrs., and 3490 hrs.

How much Evidence (and Confidence) does this sample provide for the hypothesis that $R(1200 \text{ hrs.}) \geq .99$?

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SOLUTION

From the data we find

$$\overline{X} = 2793.33 \text{ hrs.} \quad ; \quad S = 566.20 \text{ hrs.}$$

$$\alpha_3 = -0.25 \quad \text{(estimated from a Weibull slope of 5)}$$

$${}_{-.25}t_{.01} = -2.14$$

$$E = 1.8138\sqrt{6}\left(-2.14 + \frac{2793.33 - 1200}{566.20}\right) = \underline{2.995}$$

$$\text{CONFIDENCE} = \frac{1}{1 + e^{-E}} = \underline{.952}$$

UPPER LIMIT COMPLIANCE EXAMPLE USING MEAN , STANDARD DEV. & SKEWNESS

PROBLEM

Suppose the emission standard for NO_x is 2.0 grams per mile. A dozen vehicles of a certain model yield the following NO_x emission levels :

.87 g/mi , .93 g/mi , .97 g/mi , .99 g/mi , 1.02 g/mi , 1.11 g/mi ,
1.21 g/mi , 1.27 g/mi , 1.29 g/mi , 1.35 g/mi , 1.47 g/mi , 1.61 g/mi

How much confidence does this data provide for the hypothesis $R(2.0) \geq .999$?

SOLUTION

From the data we find

$$\overline{X} = 1.17417 \ ; \quad S = .23118 \ ; \quad \alpha_3 = -.28 \quad \text{(from } b=5.6\text{)}$$

$${}_{-.28}t_{.999} = +2.7$$

$$\therefore E = 1.8138\sqrt{12}\left(\frac{2 - 1.17417}{.23118} - 2.7\right) = \underline{5.48} \qquad \text{CONF.} = \frac{1}{1 + e^{-E}} = \underline{.995}$$

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SPECIAL PROBLEM 1
an AN UPPER LIMIT COMPLIANCE PROBLEM WITH NOISE

Fourteen trucks were checked for noise level as they were driven past a measuring station. The noise levels in numerical order of decibels were as follows :

TRUCK NO.	NOISE LEVEL (db)
1	61 db
2	63 db
3	65 db
4	66 db
5	67 db
6	68 db
7	68.5 db
8	69 db
9	70 db
10	70.5 db
11	71 db
12	71.5 db
13	72 db
14	74 db

If the maximum allowable noise level is 80 db , how confident can the manufacturer be that at least 99.99% of this truck population will comply ?

SOLUTION

By plotting the 14 noise measurements at their median ranks on Weibull paper, we find that the Weibull parameters are $b = 21.22967$; $\theta = 69.97283$.

(using (Use the Logarithmic Parametric Confidence)

($X_o = 80$ db) ; $R = .9999$ (Desired) ; $N = 14$; I(INTCP = -90.18591)

$$\rho_N = \left(\frac{X_N}{X_o}\right) = \frac{74}{80} = .925$$

Hence, The Logarithmic Parametric Evidence is

$$E = 1.8138\sqrt{N}\left\{\ln\ln\left(\frac{N+.4}{.7}\right) - \ln\ln\frac{1}{1-R} - b\ln\rho_N\right\}$$

$$= 1.8138\sqrt{14}\left\{\ln\ln\frac{14.4}{.7} - \ln\ln 10000 - 21.22967\ln(.925)\right\}$$

$$= 3.67373$$

Therefore, the Logarithmic Parametric Confidence is $C = \frac{1}{1+e^{-E}} = \underline{\underline{.97525}}$

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October , 1976
Page 7

SPECIAL PROBLEM 2

AN UPPER LIMIT PROBLEM IN BRAKING

A sample of six vehicles was tested for stopping distance at a specified speed at start of braking. The result were as follows : (arranged in numerical order)

VEHICLE NO.	STOPPING DISTANCE (ft)
1	23.5 ft.
2	25.0 ft.
3	25.8 ft.
4	26.3 ft.
5	27.0 ft.
6	27.8 ft.

For what stopping distance X_o can we guarantee that $R_{.95}(X_o) \geqslant .9999$?
(Use the formula for logarithmic parametric evidence)

SOLUTION

$C = .95$; $N = 6$; R(DESIRED) = .9999

$$E = \ln\left(\frac{C}{1-C}\right) = \ln\left(\frac{.95}{.05}\right) = \ln 19 = 2.94444$$

From the Data : b = Weibull slope = 17.82103 ; (INTCP = -58.46874)

θ = Characteristic Value = 26.59929 ft.

The formula for logarithmic parametric evidence is

$$E = 1.8138\sqrt{N}\left\{\ln\ln\left(\frac{N+.4}{.7}\right) - \ln\ln\left(\frac{1}{1-R}\right) - b\ln \rho_N\right\}$$

From this :

$$\ln \rho_N = \frac{1}{b}\left[\ln\ln\left(\frac{N+.4}{.7}\right) - \ln\ln\left(\frac{1}{1-R}\right) - \frac{E}{1.8138\sqrt{N}}\right]$$

$$= \frac{1}{17.82103}\left[\ln\ln\left(\frac{6.4}{.7}\right) - \ln\ln 10,000 - \frac{2.94444}{1.8138\sqrt{6}}\right] = -.117205$$

$$\rho_N = e^{-.117205} = .889403 = \frac{X_N}{X_o} = \frac{27.8}{X_o}, \therefore X_o = \frac{27.8}{.889403} = \underline{\underline{31.257\text{ft.}}}$$

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Page 8

COMPLIANCE DECISION CRITERIA

$$\left(\begin{array}{c}\text{Odds Required}\\ \text{in Favor of}\\ \text{Compliance}\end{array}\right) = \left(\frac{\text{Dollar Loss Due to Non-Compliance}}{\text{Dollar Gain When in Compliance}}\right) \times \left(\begin{array}{c}\text{Odds Desired}\\ \text{for Long -}\\ \text{Term Gains}\end{array}\right)$$

$$\left(\begin{array}{c}\text{Odds in Favor}\\ \text{of}\\ \text{Long Term Gains}\end{array}\right) = \frac{(\text{Odds in Favor of Compliance}) \times (\text{Dollar Gain When in Compliance})}{\text{Dollar Losses Due to Non-Compliance}}$$

$$\left[\begin{array}{c}\text{Evidence}\\ \text{Required}\\ \text{in Favor}\\ \text{of}\\ \text{Compliance}\end{array}\right] = \ln\left(\frac{\text{Dollar Loss Due to Non-Compliance}}{\text{Dollar Gain When in Compliance}}\right) + \left[\begin{array}{c}\text{Evidence}\\ \text{Desired}\\ \text{for}\\ \text{Long Term}\\ \text{Gains}\end{array}\right]$$

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