Statistical Bulletin
Reliability & Variation Research

DETROIT RESEARCH INSTITUTE
21900 GREENFIELD ROAD • OAK PARK, MICHIGAN 48237 • (313) 968-1818

LEONARD G. JOHNSON
EDITOR

WANG H. YEE
DIRECTOR

Vol. 5
Bulletin 2

May, 1975
Page 1

SIMPLIFIED WEIBULL COMPARISON CONFIDENCE THEORY --- (AT ANY DESIRED LOWER QUANTILE)

INTRODUCTION : Exact Comparison Confidence Formulas at the First Order Statistic Level and at the Mean Level.

Assume two neighboring samples, each of size N and of the same Weibull slope b .

Let θ_1 = Characteristic Value of Sample #1 (on the left)

Let θ_2 = Characteristic Value of Sample #2 (on the right)

This is represented graphically in FIGURE 1 below :

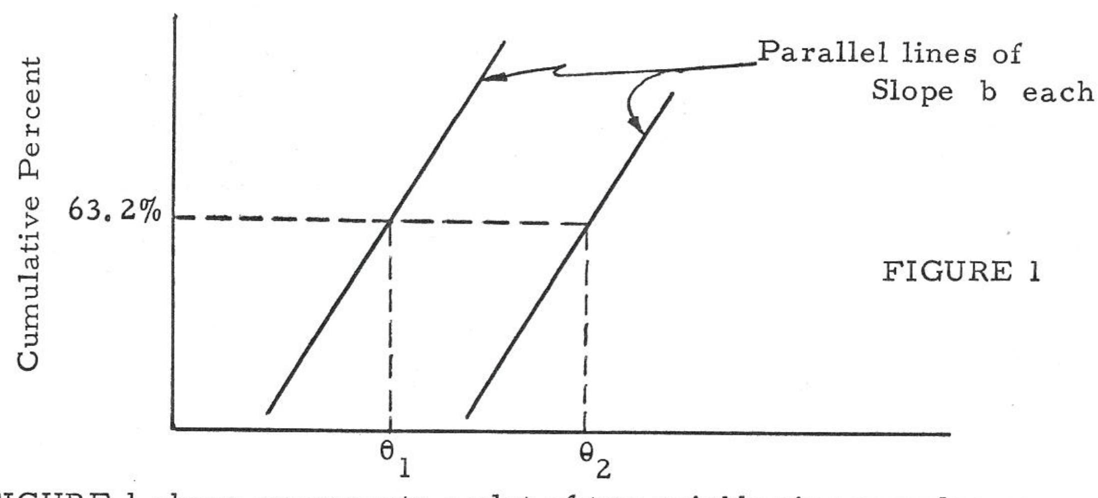

FIGURE 1

NOTE: FIGURE 1 above represents a plot of two neighboring samples on Weibull Probability Paper.

Published 8 times a year by the Detroit Research Institute. Subscription Rate: One year $32.00. Quantity rates on request.
Address subscription orders and editorial correspondence to:
Detroit Research Insitute, 21900 Greenfield Road, Oak Park, Michigan 48237

At $B_{\frac{.7}{N+.4}}$ level: $C = \dfrac{1}{1 + \left(\dfrac{\theta_2}{\theta_1}\right)^{-b}} = \dfrac{1}{1 + \rho^{-b}}$

NOTE: $\dfrac{.7}{N + .4}$ = Median Rank of 1st in N

ρ = Characteristic Value Ratio = $\dfrac{\theta_2}{\theta_1}$

C = Confidence that (2) is better than (1)

AT MEAN Level; $C = \dfrac{1}{1 + \left(\dfrac{\theta_2}{\theta_1}\right)^{-b\sqrt{N}}} = \dfrac{1}{1 + \rho^{-b\sqrt{N}}}$

Let Q_1 = Quantile Level of 1st in N = $\dfrac{.7}{N + .4}$

Let Q_M = Quantile Level of the Mean = $1 - e^{-\left[\Gamma\left(1 + \frac{1}{b}\right)\right]^b}$

Then:

At B_{Q_1}: Confidence (of a difference) = $C_1 = \dfrac{1}{1 + \rho^{-b}}$

At B_{Q_M}: Confidence (of a difference) = $C_M = \dfrac{1}{1 + \rho^{-b\sqrt{N}}}$

(NOTE: These formulas are exact.)

II EMPIRICAL EXTENSION :

At the quantile level of the j^{th} in N , i.e., at B_{Q_j} , define the Comparison Confidence by the formula

$$C_j = \frac{1}{1 + e^{-b\sqrt{2j - 1}}} \qquad (1)$$

This gives $C_1 = \dfrac{1}{1 + e^{-b}}$ for $j = 1$,

and gives $C_M = \dfrac{1}{1 + e^{-b\sqrt{N}}}$ for $j = \dfrac{N + 1}{2}$

Thus, by means of the Empirical Extension Formula (1) above, We have maaged to find a way of interpolating between $j = 1$ and $\dfrac{N + 1}{2}$.

III REVISING THE EMPIRICAL EXTENSION FORMULA IN TERMS OF QUANTILE LEVEL Q AND TOTAL DEGREES OF FREEDOM T.

The quantile level Q for the j^{th} value in N has the Median Rank Formula

$$Q = \frac{j - .3}{N + .4}$$

Hence,

$$j = .3 + Q(N + .4)$$

$$\therefore 2j - 1 = 2Q(N + .4) - .4$$

$$\approx 2QN$$

$$\approx 2QT^{1/2}$$

$$T = \text{Total Degrees of Freedom} = (N - 1)^2$$

Therefore, a simplified CONFIDENCE FORMULA at the Quantile Level Q is

$$C_Q = \frac{1}{1 + e^{-bT^{1/4}\sqrt{2Q}}} \qquad (0 \leq Q \leq .5)$$

(2)

IV COMPARISON CONFIDENCE FORMULAS FOR SPECIAL LOWER QUANTILE Q

ρ = Observed Life Ratio (at Quantile Q)

b = Weibull Slope

T = Total Degrees of Freedom = $(N_1 - 1)(N_2 - 1)$

N_1 = Size of Sample #1 (On the left)

N_2 = Size of Sample #2 (On the right)

WARNING: ρ is equal to $\left(\frac{\theta_2}{\theta_1}\right)$ only when the Weibull slopes are equal.

At Q = .01 : $$C = \frac{1}{1 + \rho^{-.14142\, b\, T^{1/4}}}$$

At Q = .05 : $$C = \frac{1}{1 + \rho^{-.31623\, b\, T^{1/4}}}$$

At Q = .1 : $$C = \frac{1}{1 + \rho^{-.44721\, b\, T^{1/4}}}$$

At Q = .2 : $$C = \frac{1}{1 + \rho^{-.63246\, b\, T^{1/4}}}$$

At Q = .3 : $$C = \frac{1}{1 + \rho^{-.77460\, b\, T^{1/4}}}$$