Statistical Bulletin
Reliability & Variation Research

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Vol. 5 — April, 1975
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COST ORIENTED RELIABILITY

I: QUESTION: HOW MUCH TESTING NEEDS TO BE DONE ?

(a) This question cannot be answered until we first answer the following question :

"What minimum reliability is required ?"

1. The required minimum reliability depends on two basic factors. These are : RISK and OPPORTUNITY.

RISK

RISK is defined as the MONETARY LOSS resulting from the FAILURE of a product (each time it fails), multiplied by the number sold, and, also, multiplied by the PROBABILITY of an individual failure before warranty expiration. Thus,

Let N = Number of Sales Expected

Let E_f = Expense of Repairing (or replacing) an Item which fails before target

Let C_f = Probability of Failure (before target)

Then RISK = $N C_f E_f$

OPPORTUNITY

OPPORTUNITY is defined as MONETARY GAIN resulting from the SUCCESS of a sold individual item , multiplied by the number sold , and , also, multiplied by the PROBABILITY OF SUCCESS of a single item (to target).

Thus

Let N = Number of Sales Expected

Let S_r = Selling Price of a Reliable (successful) item

Let C_r = Probability of Success (reliability) to Target

Then OPPORTUNITY $= N S_r C_r$ (If customer returns item and gets full refund in case of failure)

$= N S_r$ (If customer accepts repair in case of failure)

In answer to the question of "How much Reliability is required?" , we simply stipulate that there must be enough reliability to produce a RETURN of P % on our INVESTMENT . This requires that the following DECISION EQUATION is satisfied :

$$\text{OPPORTUNITY} - \text{RISK} = (1 + P/100) * \text{INVESTMENT} \qquad (1)$$

EXAMPLE

We are thinking of going into the business of selling a certain design, which involves an investment totalling \$ 2,500,000.
We expect to sell 1000 such designs, and we expect to realize a 10% return on our investment. Each design sells for \$3000. Each repaired failure will cost \$4000. What Reliability is required of the design ?

ANALYSIS OF RISK

We have $N = 1000$

$C_f = 1 - R$ (R = RELIABILITY)

$E_f = \$4000$

$$\text{RISK} = 4,000,000\,(1 - R)$$

ANALYSIS OF OPPORTUNITY

We have $N = 1000$

$C_r = R$ (R = RELIABILITY)

$S_r = \$3000$

$$\text{OPPORTUNITY} = 3,000,000$$ (Assuming customer accepts repair in case of failure)

According to the DECISION EQUATION (1) :

OPPORTUNITY - RISK = (1 + 10/100) (2,500,000)

Thus , $3,000,000 - 4,000,000\,(1 - R) = 1.1 \times 2,500,000$

or , $3,000,000 - 4,000,000 + 4,000,000\,R = 2,750,000$

or , $4,000,000\,R = 3,750,000$

$$R = 0.9375 \quad \text{(Minimum)}$$

To guarantee a MINIMUM RELIABILITY of 0.9375 , (say, with 90% confidence) , we need to test N_t items (to target) without a single failure, where

$$N_t = \frac{\log(1 - C)}{\log R} \qquad \left(\begin{array}{l} C = .9 \\ R = .9375 \end{array}\right)$$

$$= \frac{\log .1}{\log .9375}$$

$$= \frac{-1}{-.02803}$$

$$= \underline{\underline{36}} \qquad \text{(Answer)}$$

Thus , we should carry out a test program in which we test 36 of these items to the warranty target without any of them failing the test.

ANOTHER TYPICAL EXAMPLE

The adoption of a new plastic part in an assembly involves an investment of THIRTY MILLION DOLLARS. The plastic part sells for \$5.00, and we have seven million customers using this assembly. It will cost us \$7.00 each time the part must be replaced under warranty.

(a) How reliable must the part be in order to guarantee a 15% return on our investment with 90 % confidence ?

(b) How many parts must be tested to the warranty target life without any failures ?

(a) $$R = 1 + \frac{\left(1 + \frac{15}{100}\right)(30 \times 10^6)}{(7 \times 10^6) \times 7} - \frac{5}{7} = 0.9898$$

GENERAL FORMULA FOR SAMPLE SIZE WITH ZERO FAILURES

(b) $$N = \frac{\log(1 - C)}{\log\left[1 + \frac{\left(1 + \frac{P}{100}\right) I}{T\,E_f} - \frac{S_r}{E_f}\right]} = \frac{\log(1 - C)}{\log R}$$

$$N = \frac{\log(1 - .9)}{\log\left[1 + \frac{(1 + 15/100)(30 \times 10^6)}{7 \times 10^6 \times 7} - \frac{5}{7}\right]} = \underline{225\ (\text{Ans.})}$$

N = Sample size (with zero failures)

C = Confidence

P = Percent return on investment

I = Investment

T = Total Sales

S_r = Selling Price per unit

E_f = Repair Expense per failed unit

NOTE : It is assumed that the customer accepts repair in case of failure.

EXTENSION OF THE CONCEPTS TO COMPARATIVE LIFE TESTING

Suppose we have the situation depicted in FIGURE 1 below :

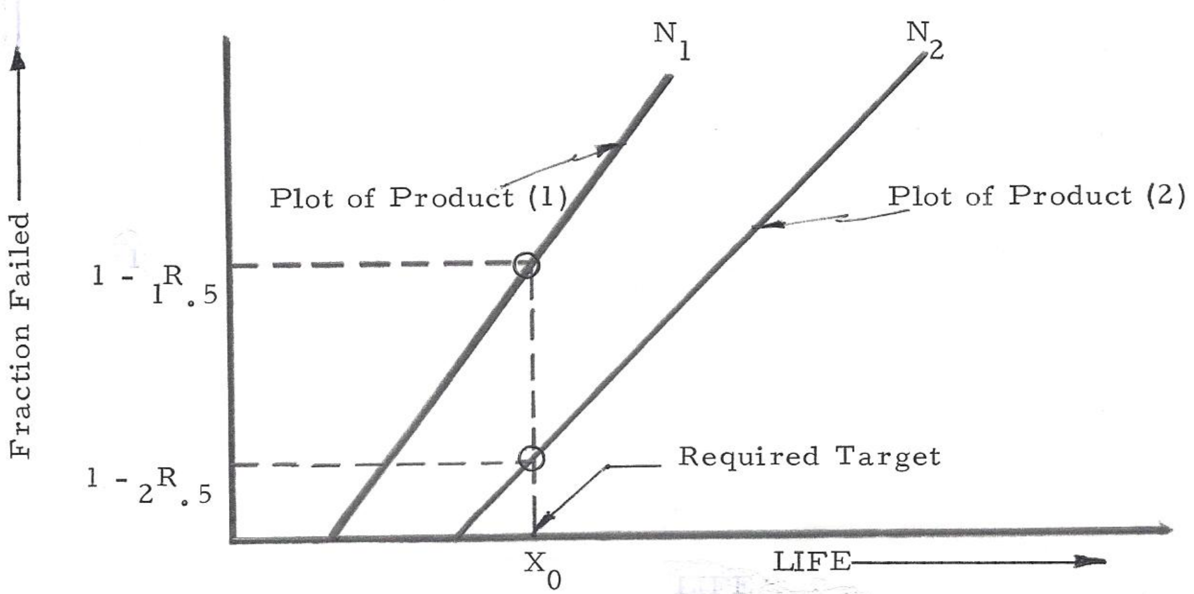

FIGURE 1

In FIGURE 1 , product (1) has a Weibull plot based on a sample of N_1 failures , while product (2) (which is supposely better than product (1)) has a Weibull plot based on a sample of N_2 failures. We now raise the following question :

QUESTION : How large should the sample sizes N_1 and N_2 be in order to assure Δ percent more return on our investment in product (2) than our percent return on investment in product (1) , with , say, 90% confidence ?

DECISION ANALYSIS

Each product has its own DECISION EQUATION . The two Decision Equations are as follows :

$$\left.\begin{array}{ll} \text{For (1):} & \text{OPPORTUNITY}_1 - \text{RISK}_1 = (1 + P/100) * (\text{INVESTMENT}_1) \\ \text{For (2):} & \text{OPPORTUNITY}_2 - \text{RISK}_2 = \left(1 + \frac{P + \Delta}{100}\right) * (\text{INVESTMENT}_2) \end{array}\right\}$$

In terms of WARRANTY REPAIR COSTS, SELLING PRICES, PERCENT RETURN , and INVESTMENTS , and TOTAL SALES , these decision equations become :

$$\left.\begin{array}{ll} (1): & T_1\, {}_1S_r\, {}_1C_r - T_1\, {}_1C_f\, {}_1E_f = (1 + P/100)\, I_1 \\ (2): & T_2\, {}_2S_r\, {}_2C_r - T_2\, {}_2C_f\, {}_2E_f = \left(1 + \frac{P + \Delta}{100}\right) I_2 \end{array}\right\}$$

NOTE : In Figure 1, the symbol ${}_1R_{.5}$ denotes the RELIABILITY of product (1) to target X_0 with 50 % confidence, while ${}_2R_{.5}$ denotes the RELIABILITY of product (2) to the same target X_0 with 50% confidence. It should be borne in mind that these Weibull plots are MEDIAN RANK PLOTS, and , consequently, all points on them represent the 50% confidence level, since MEDIAN RANKS ARE 50 % RANKS.

Putting ${}_1C_r$ = R_1 = Reliability of Product (1)

${}_1C_f$ = $1 - R_1$ = Failure Probability of Product (1)

${}_2C_r$ = R_2 = Reliability of Product (2)

${}_2C_f$ = $1 - R_2$ = Failure Probability of Product (2)

and , letting $T_1 = T_2 = T$ (Total Sales)

${}_1E_f = {}_2E_f = E_f$ (Unit Rpair Expense)

${}_1S_r = {}_2S_r = S_r$ (Unit Selling Price)

$I_1 = I_2 = I$, we derive the following

IMPROVEMENT DIFFERENCE EQUATION :

$$R_2 - R_1 = \frac{I \Delta}{100\ T\ E_f} = \delta$$

(Assuming customer accepts repair in case of failure)

In plain English , this improvement decision equation says that THE IMPROVEMENT IN RELIABILITY IS EQUAL TO THE IMPROVEMENT IN PERCENT RETURN ON INVESTMENT MULTIPLIED BY THE FACTOR

$\frac{I}{100\ T\ E_f}$, WHERE

I = Total Investment

T = Total Sales

E_f = Cost Per Warranty Repair (or replacement)

S_r = Selling Price Per Unit

Conversely, we can solve for Δ (the improvement in percent return) in terms of the RELIABILITY IMPROVEMENT δ :

Thus ,

$$\Delta = \frac{100\ T\ E_f\ \delta}{I}$$

In order to realize 90% confidence in Δ (the change in percent return) we must put δ (the reliability change) at the 90% confidence level.

The 50% confidence level of δ is $\delta_{.50} = {}_2R_{.50} - {}_1R_{.50}$,

where ${}_1R_{.50}$ and ${}_2R_{.50}$ are the OBSERVED RELIABILITIES on the MEDIAN RANK WEIBULL PLOTS OF FIGURE 1 . We call $\delta_{.50}$ the MEDIAN OBSERVED RELIABILITY IMPROVEMENT in going from PRODUCT (1) to PRODUCT (2).

According to the LOGISTIC CONFIDENCE FORMULA, we can write the CONFIDENCE OF A RELIABILITY DIFFERENCE δ_C as follows :

$$C = \frac{1}{1 + e^{-\frac{2.565\,(\delta_{.50} - \delta_C)}{\sigma_1 + \sigma_2}}}$$

where C = Confidence

$$\sigma_1 = \sqrt{\frac{{}_1R_{.50}\,(1 - {}_1R_{.50})}{N_1}}\ , \text{ and } \sigma_2 = \sqrt{\frac{{}_2R_{.50}\,(1 - {}_2R_{.50})}{N_2}}$$

Solving the LOGISTIC CONFIDENCE FORMULA for δ_C yields

$$\delta_C = \delta_{.50} - \left(\frac{\sigma_1 + \sigma_2}{2.565}\right)\left(\ln \frac{C}{1 - C}\right)$$

Putting C = .90 , this becomes

$$\delta_{.90} = \delta_{.50} - .85662\,(\sigma_1 + \sigma_2)$$

Using this value of $\delta_{.90}$, we find that the GUARANTEED INCREASE IN PERCENT RETURN ON INVESTMENT $\Delta_{.90}$ (with 90% confidence) is

$$\Delta_{.90} = \frac{100\ T\ E_f \left[\delta_{.50} - .85662(\sigma_1 + \sigma_2)\right]}{I}$$

Solving this for the required sample size N yields

$$N = \left[\frac{.85662\left(\sqrt{R_1(1 - R_1)} + \sqrt{R_2(1 - R_2)}\right)}{\delta_{.50} - \dfrac{I\,\Delta_{.90}}{100\ T\ E_f}}\right]^2$$

(Assuming the customer accepts repair in case of failure)

where, for brevity,

R_1 denotes the OBSERVED MEDIAN RELIABILITY ${}_1R_{.50}$

and R_2 denotes the OBSERVED MEDIAN RELIABILITY ${}_2R_{.50}$.

NUMERICAL EXAMPLE IN COMPARATIVE LIFE TESTING

PROBLEM : A new product involves a One hundred fifty million dollar investment. It is to replace a former product which had a reliability of 94% to the service target. This new product has a reliability of 99% to the same service target. Also, the following relevent figures are given :

Total Sales = 1, 000, 000 units

Cost per Warranty Repair = $260

Selling Price per unit = $195

Desired Increase in Percent Return = 1%

(It is assumed that the customer accepts repair in case of failure).

QUESTION : What sample size of each product (former and new) must be tested to failure for the two Weibull plots which are expected to show OBSERVED MEDIAN RELIABILITIES of .94 and .99 , respectively, for the former product and the new (more reliable) product, in order to be 90% confident that the percent return on investment will be at least 1% more ?

SOLUTION

We have , ${}_1R_{.50} = .94$, and ${}_2R_{.50} = .99$ $\left(\therefore \delta_{.50} = .05\right)$

$\Delta_{.90} = 1$

$E_f = \$260$ and $S_r = \$195$

$T = 1,000,000$ units , and $I = \$150,000,000$

Substituting these values into the SAMPLE SIZE FORMULA on page 10 we obtain

$$N = \left[\frac{.85662 \sqrt{.94(.06)} + \sqrt{.99(.01)}}{.05 - \dfrac{150 \times 10^6 \times 1}{100 \times 10^6 (260)}} \right]^2 = \underline{43}$$

Thus, our two Weibull plots (for the former and the new) must both have sample size at least 43 and show that the Observed Reliabilities are .94 and .99 , respectively.