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HOW TO COMBINE INDEPENDENT COMPONENT RELIABILITIES, AS WELL AS COMBINING THE CONFIDENCE LEVELS OF THE COMPONENT RELIABILITIES INTO A RESULTANT CONFIDENCE FOR THE SYSTEM RELIABILITY

INTRODUCTION

In the broad field of product reliability it is important to be thoroughly trained in all aspects of the subject. Otherwise, there is the danger of making false assumptions which can lead us astray. One such false assumption, which we shall discuss in this bulletin, has to do with the multiplication of two independent component reliabilities making up a system, as illustrated in *Figure 1* below:

Figure 1

In Figure 1 we have component I in series with component II to form the system from A to B.

Suppose it is desired that the system survives x_o hours of operation.

Let ${}_{I}R(x_o)$ = Reliability of component I to x_o hours

Let ${}_{II}R(x_o)$ = Reliability of component II to x_o hours

Let ${}_{AB}R(R_o)$ = Reliability of the system AB to x_o hours

$${}_{AB}R(x_o) = {}_{I}R(x_o) \cdot {}_{II}R(x_o)$$

i.e., the system reliability is the product of the two independent component reliabilities.

There is no special difficulty with this relationship if we have the actual true reliabilities of I and II with 100% confidence. Then we would be 100% confident that

$$_{AB}R(x_o) = {}_{I}R(x_o) \cdot {}_{II}R(x_o)$$

But, now suppose $_{I}R(x_o)$ is at 90% confidence, and, likewise $_{II}R(x_o)$ is at 90% confidence. What would be the correct confidence level for $_{AB}R(x_o)$?

We have seen discussions in which it was assumed that if we have $_{I}R_{.90}(x_o)$ and $_{II}R_{.90}R(x_o)$, i.e., both component reliabilities at 90% confidence, then the product of $_{I}R_{.90}(x_o) \cdot {}_{II}R_{.90}(x_o)$ would yield $_{AB}R_{.90}(x_o)$, that is the system reliability would also be at 90% confidence. This, unfortunately, is an incorrect conclusion.

The handling of this question regarding the confidence level for $_{AB}R(x_o)$ is clarified by a discussion between a professional probability expert and a beginner in reliability analysis.

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A PROFESSIONAL RELIABILITY EXPERT CLARIFIES A PUZZLING SITUATION

EXPERT: I have a very important question about your work in reliability analysis, Mr. Beginner.

BEGINNER: What is your question?

EXPERT: It has to do with the question of confidence level to be assigned to the system reliability for a system which consists of two independent components in series, when we know that each component reliability is at 90% confidence.

BEGINNER: Wouldn't the system reliability obtained by multiplying the two independent component reliabilities also be at 90%?

EXPERT: You are being too pessimistic about the confidence for the system reliability when you assume it is only 90%.

BEGINNER: Then, how should this situation be correctly handled?

EXPERT: What you must do is get the resultant odds for the system reliability by multiplying the odds for the individual component reliabilities.

BEGINNER: How do I do that?

EXPERT: You take 90% confidence for the first component reliability and convert it to 9:1 odds, i.e., 90% in favor, and 10% against, which yields odds of 90/10 = 9/1. Likewise, the second component reliability at 90% confidence has odds 9/1. Then, to obtain the resultant odds for the system reliability we multiply the two component odds to obtain

$$\text{Resultant Odds} = (9/1)\cdot(9/1) = 81/1 .$$

Thus, we have odds of 81 to 1 in favor of the system reliability given by

$${}_{\text{System}}R = {}_{I}R \cdot {}_{II}R .$$

81 to 1 odds implies a resultant confidence of 81/82 = 0.9878. So, for a desired target life x_o we have

$$ {}_{I}R_{.90}(x_o) \cdot {}_{II}R)x_o) = {}_{System}R_{.9878}(x_o) $$

So, you see, Mr. Beginner, the resultant confidence for the system reliability is much more than the 90% confidence indices you had for the individual component reliabilities.

BEGINNER: That's very interesting. You are telling me that if, in general, I have ${}_{I}R_{c1}(x_o)$ and ${}_{II}R_{c2}(x_o)$ as my component reliabilities with confidence indices of c_1 and c_2 respectively, then I must convert c_1 into *odds* $c_1/(1- c_1)$ and, also convert confidence c_2 into *odds* $c_2/(1 - c_2)$ and then multiply $c_1/(1- c_1) \cdot c_2/(1 - c_2)$ to obtain the *resultant odds* for the system reliability.

EXPERT: That's right. Now you have caught on the right approach.
For example, suppose we have ${}_{I}R_{.90}(x_o)$ and ${}_{II}R_{.95}(x_o)$, and we multiply them to get

$$ {}_{System}R_{\hat{c}}(x_o) = {}_{I}R_{.90}(x_o) \cdot {}_{II}R_{.95}(x_o) \quad . $$

Then the resultant odds would be

$$ \frac{\hat{c}}{1 - \hat{c}} = \left(\frac{.90}{.10}\right)\left(\frac{.95}{.05}\right) = 9 \cdot 19 = 171 \quad \text{to} \quad 1 \quad . $$

Consequently, $\hat{c} = 171/172 = 0.9942$

So, the system reliability obtained by multiplying the two component reliabilities would be at a confidence level of 99.42% , which is greater than either the 90% confidence for the reliability of component I or the 95% confidence for the reliability of component II.

BEGINNER: So, in general, when individual component reliabilities have confidence levels above 50% each, it follows that the resultant system reliability obtained by multiplying the component reliabilities is a bit more than either of the separate confidence levels of the two component reliabilities.

EXPERT: That's right. In general, after you have converted confidence indices to odds and then multiplied these odds, you will have a resultant odds $\hat{O} = O_1 O_2$,

where $\hat{O}$ = Resultant Odds on the System Reliability
O_1 = Odds on Reliability of Component I
O_2 = Odds on Reliability of Component II

Then, the Resultant Confidence is

$$\hat{C} = \frac{\hat{O}}{1 + \hat{O}}$$

CONCLUSION

We have illustrated how the proper way of combining confidence indices for independent reliabilities in series leads to a much higher confidence level for the system reliability than either of the confidence indices for the reliabilities of the two components when both component reliabilities have confidence indices above 50%.

A table for some specific confidence level combinations is given below:

CONFIDENCE FOR $_I\underline{R}$	CONFIDENCE FOR $_{II}\underline{R}$	CONFIDENCE FOR $_{SYS}\underline{R}$
.50	.50	.50
.60	.60	.6923
.70	.70	.8448
.80	.80	.9412
.90	.90	.9878
.95	.95	.9974
.99	.99	.9999