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HOW TO USE LOG-LOG GRAPH PAPER TO MAKE LINEAR ENTROPY PLOTS OF LIFE TEST DATA WITH A FINITE MAXIMUM LIFE

INTRODUCTION

We have found that straight line plots on log-log graph paper theoretically represent the growth of entropy (failures per system) in cases where life test data have a Weibull Distribution. This is all fine and dandy as long as it is practical to assume that the maximum life is infinite. However, in cases of finite maximum life, it becomes necessary to modify the horizontal abscissa axis on the log-log graph in order to keep the entropy growth plot linear.

The way to make this proper modification is to make the abscissa at life x represent the quantity defined by the formula

$$\zeta = \frac{\text{Fraction of Range Below Life x}}{\text{Fraction of Range Above Life x}}$$

where the *range* is defined as the entire difference between the maximum life and the minimum life.

Thus, if A = Minimum Life and if U = Maximum Life, then at any life x, we define the modified to be

$$\zeta = \frac{x - A}{U - x}$$

In case of *Zero Minimum Life* we have A = 0, and then the newly defined abscissa is

$$\zeta = \frac{x}{U - x}$$

THE APPROPRIATE ENTROPY FUNCTION WITH FINITE MAXIMUM LIFE

Since every failure process has a specific type of *Entropy Function* which defines the rate of deterioration or failure rate per active item versus elapsed service time, it becomes necessary to establish such an entropy function in the case of a finite maximum life. The entropy function must be flexible enough to give different variabilities in the life distribution, ranging from small variabilities (with a steep slope on log-log paper) to large variabilities (with a small slope on log-log paper). The most reasonable mathematical entropy function which meets these general requirements is a power function of the form

$$\varepsilon(x) = K\left(\frac{x - A}{U - x}\right)^{m}$$

where,
- m = Slope of Linear Plot on Log-Log Paper
- K = Entropy at Mid-Range, i.e., at x = 0.5(A + U)
- A = Minimum Life
- U = Maximum Life

In case the minimum life A = 0, the above entropy function becomes

$$\varepsilon(x) = K\left(\frac{x}{U - x}\right)^{m}$$

By taking the natural logarithms of both sides we obtain

$$\ln \varepsilon(x) = m \ln\left(\frac{x}{U - x}\right) + \ln K$$

Thus, by plotting $\frac{x}{U-x}$ as our modified abscissa, and ***Entropy*** as ordinate on log-log paper, we obtain a line of slope **m** with the entropy value **K** at mid-range, where x = U/2.

AN ACTUAL EXAMPLE OF LIFE TEST DATA WITH A FINITE MAXIMUM LIFE

Suppose we run a life test on a sample of 10 units, with the following results:

TABLE 1

Failure No.	Hrs. to Failure	Median Rank	Entropy
1	121	0.0673	0.0697
2	250	0.1635	0.1785
3	352	0.2596	0.3006
4	433	0.3558	0.4397
5	470	0.4519	0.6013
6	491	0.5481	0.7943
7	502	0.6442	1.0334
8	508	0.7404	1.3486
9	512	0.8305	1.8109
10	514	0.9327	2.6986

We plot these initially on Weibull graph paper. This yields Figure 1, in which we estimate a Maximum Life of 520 hours.

Next we plot on log-log graph paper by taking the abscissa corresponding to each life x to be

$$\zeta = \frac{\text{Fraction of Range Below Life x}}{\text{Fraction of Range Above Life x}}$$

In this case, the range is 520 hours. The entropies corresponding to the fractions failed (median ranks) are taken as ordinates on log-log paper. So we now have the following tabulation:

TABLE 2

Hours to Failure	(Abscissa) - ζ	Entropy
121	121/399=0.3033	0.0697
250	250/270=0.9259	0.1785
352	352/168=2.6952	0.3006
433	433/87=4.9770	0.4397
470	470/50=9.4000	0.6013
491	491/29=16.9310	0.7943
502	502/18=27.8880	1.0334
508	508/12=42.3333	1.3486
512	512/8=64.0000	1.8109
514	514/6=85.6667	2.6986

This table of data gives us the linear log-log plot shown in Figure 2.

FIGURE 1

WEIBULL PLOT OF SAMPLE DATA IN TABLE 1

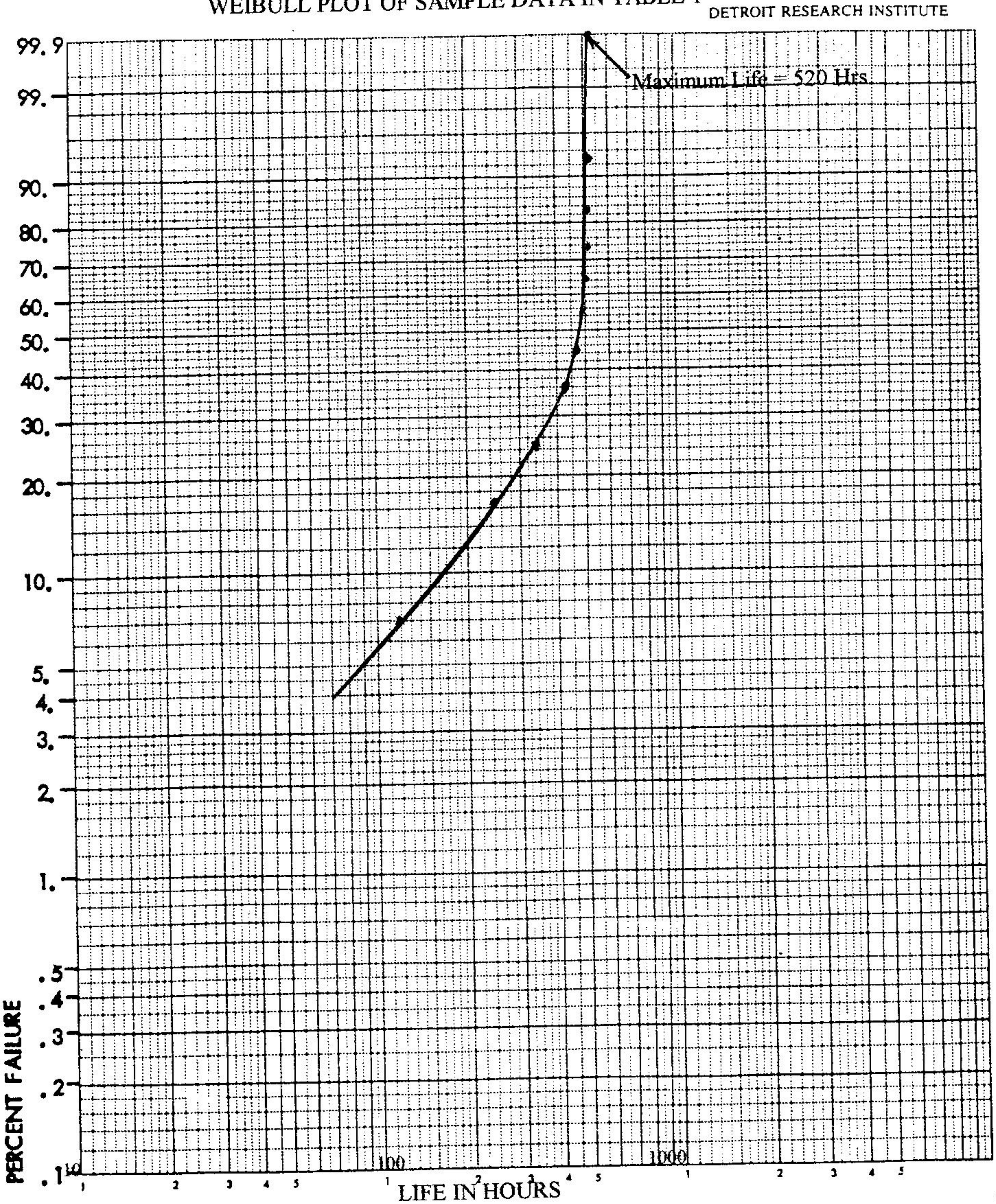
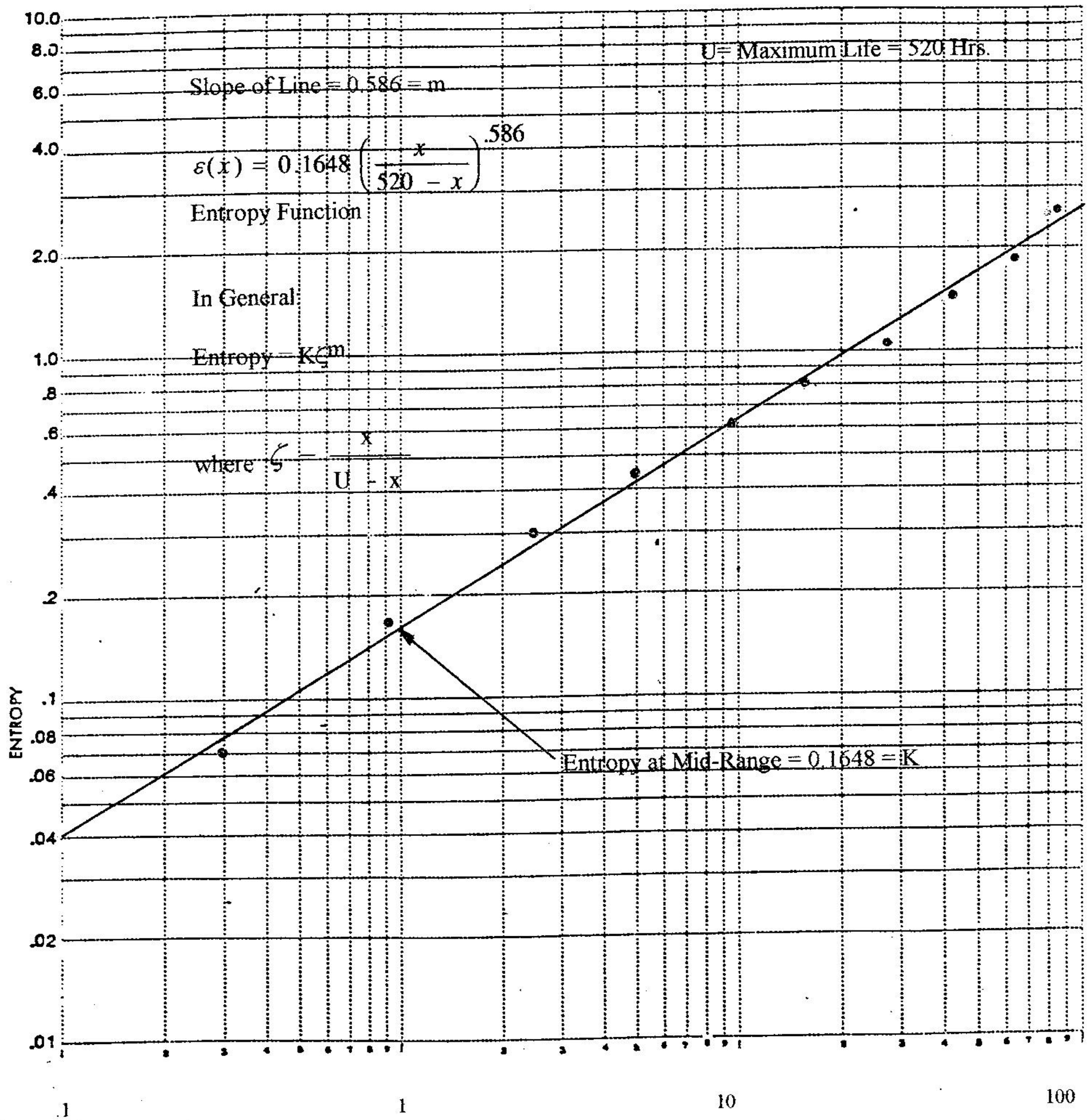


FIGURE 2

LOG-LOG PLOT OF DATA IN TABLE 2



$$\zeta = \frac{\text{Fraction of Range Below Life x}}{\text{Fraction of Range Above Life x}}$$

CONCLUSION

We have demonstrated that a usable entropy function for life test data with a finite maximum life is of the form

$$\varepsilon(x) = K\left(\frac{x}{U - x}\right)^{m}$$

Then, by taking a transformed abscissa as $\zeta = \dfrac{x}{U - x}$

$= \dfrac{\text{Fraction of Range Below Life x}}{\text{Fraction of Range Above Life x}}$, and ordinate = Entropy,

we can get a linear plot on log-log paper with slope = m and an entropy of K at mid-range (where $\zeta = 1$).