STATISTICAL BULLETIN
Reliability & Variation Research

LEONARD G. JOHNSON
EDITOR

DETROIT RESEARCH INSTITUTE
P.O. Box 36504 • Grosse Pointe, MI 48236 • (313) 886-8435

WANG H. YEE
DIRECTOR

Volume 27
Bulletin 7

January, 1998
Page 1

DESIGNING ACCELERATED LIFE TESTS WITHOUT ANY OVERLOAD BY USING SAMPLES WHICH TAKE LESS TIME TO TEST AND STILL COMPLY WITH WHATEVER RELIABILITY AND CONFIDENCE LEVELS MIGHT BE DICTATED BY GAINS AND LOSSES AND PROFITABILITY

ACCELERATED LIFE TESTING — A DECISION PROCESS

In testing any product design we are involved in making certain decisions regarding the product's acceptability. For example, we want to know to what extent the product will perform satisfactorily for a desired number of cycles or time period. This is known as the study of compliance to a desired life goal or service target. Then the question which comes up is "To what extent do we want the product to comply with the life goal?". In other words, what fraction of all such products produced need to comply in order to make the entire business sufficiently profitable by living up to a promised life or warranty period? In pursuing such a program of compliance investigation we need to know how much is gained from selling the production total to customers, as well as losses suffered when an item fails to perform successfully to the promised warranty target. This is the *First Commandment* regarding the setting up of a life testing program, including accelerated tests. It is no use attempting to test without such a gain and loss basis, for that is just plain gambling and trusting that we'll be lucky enough to have a profitable business. We must always have a large enough index of *Confidence* to be able to realize enough total *Gains* which will more than offset our total *Losses* due to failures to comply with our warranty promises for the design being sold.

So, this means that our sample size in life testing must be large enough to yield the necessary *Confidence* of complying with our promise of *Reliability* to a warranty target. In many such testing situations it turns out that the required sample size is so large we are compelled to reduce it by employing accelerated testing plans with smaller samples or special techniques to shorten testing time to reach the desired level of *Confidence* dictated by our desired advantage of *Gains* exceeding *Losses*.

In this bulletin we shall first of all introduce the reader to the ***Economic Foundation*** of the test ***Confidence*** dictated by gains and losses, and then proceed to arrive at acceptable sample sizes and waiting times in order to be able to wisely release the tested product for sale to the public with the proper level of assurance *(Evidence)* of ending up with sales *Gains* exceeding *Losses* from bad items by acceptable *Profitable Ratio* defined as the ratio *Gains/Losses.*

The fundamental economic basis for any life testing program is the need for the appropriate level of *Odds* in favor of compliance to the life required in service. This required level of *Odds* is given by the formula

$$Odds = \frac{K \cdot L}{G}$$

where K = Desired Profitability Ratio
= Total Dollars Gained/Total Dollar Lost
and, L = Dollar Loss per Non-Complying Case
and, G = Dollar Gain per Complying Case

The required *Evidence* is then the natural logarithm of *Odds*, i.e., *ln(Odds).*

NOTE: The following BASIC computer program is for calculating the Median Values when Weibull Slope, Characteristic Value, and sample size are known. (This is for the examples on the following pages.)

```
5  CLS:COLOR 15,1
10 LOCATE 3,15:PRINT "MEDIAN VALUES WITH KNOWN SLOPE AND CHAR. VALUE PROGRAM"
20 Q=0:TV=0:PRINT
30 INPUT "WEIBULL SLOPE";B
40 INPUT "CHARACTERISTIC LIFE";T
50 INPUT "SAMPLE SIZE";N
60 PRINT
70 PRINT TAB(10);"ORDER STATISTIC NO.";"     VALUES     ";"MEDIAN RANKS"
80 FOR J=0 TO N-1
90 Q = INT(.5+T*((-LOG((N-J-.3)/(N+.4)))^(1/B)))
100 TV=TV+Q
110 PRINT TAB(18);J+1;TAB(35);Q;TAB(45);1-(N-J-.3)/(N+.4)
120 NEXT J
140 PRINT TAB(25);"TOTAL VALUES =";TV
150 END
```

EXAMPLE OF ACCELERATED LIFE TESTS--- WITH DECISION

EXAMPLE #1: In a Weibull population with Slope 2.5 and a Characteristic Life of 1,000 Hours a typical sample of ten items would have the following values:

```
MEDIAN VALUES WITH KNOWN SLOPE AND CHAR. VALUE PROGRAM

WEIBULL SLOPE? 2.5
CHARACTERISTIC LIFE? 1000
SAMPLE SIZE? 10
```

ORDER STATISTIC NO.	VALUES	MEDIAN RANKS
1	345	6.730765E-02
2	502	.1634615
3	618	.2596154
4	720	.3557692
5	816	.4519231
6	912	.548077
7	1013	.6442308
8	1127	.7403846
9	1268	.8365385
10	1487	.9326923

TOTAL VALUES = 8808

A representative sample size of 3 would have the following typical values:

```
MEDIAN VALUES WITH KNOWN SLOPE AND CHAR. VALUE PROGRAM

WEIBULL SLOPE? 2.5
CHARACTERISTIC LIFE? 1000
SAMPLE SIZE? 3
```

ORDER STATISTIC NO.	VALUES	MEDIAN RANKS
1	556	.2058824
2	864	.5
3	1201	.7941177

TOTAL VALUES = 2621

CONCLUSION: If we had test the sample of 10 one item at a time the total elapsed testing time would be 8,808 hours.

On the other hand, testing two random samples of size 3 each (one item at a time) would required a total of 5,242 hours.

The important thing to note is that two samples of 3 each would yield more Evidence than one sample of size 10. This is due the fact that Evidence is proportional to the square root of the sample size. Thus,

One sample of 10 has Evidence proportional to $\sqrt{10}$, while the Evidence from two sample of size 3 is proportional to

$2\sqrt{3} = \sqrt{12} > \sqrt{10}$.

DECISION: Test two independent samples of size 3 each rather than one sample of size 10

EXAMPLE #2

Suppose a Weibull population has a slope of 2 and a Characteristic Life of 1,000 Hours. A typical random sample of 20 items from such a population would look as follows:

ORDER STATISTIC NO.	LIFE IN HOURS	
1	187	
2	295	
3	377	
4	447	
5	512	
6	572	
7	631	
8	688	
9	748	Total Hours
10	803	----------------
11	862	17,567
12	923	
13	987	
14	1055	
15	1129	
16	1212	
17	1307	
18	1422	
19	1576	
20	1836	

For a random sample size 5 the typical values are:

ORDER STATISTIC NO.	LIFE IN HOURS	
1	373	
2	615	Total Hours
3	833	----------------
4	1075	4,326
5	1429	

Two such samples of 5 would take 8,652 hours to test one item at a time. Yet, the Evidence would be equivalent to that obtained from one sample of 20. This due to the fact that $2\sqrt{5} = \sqrt{20}$.

DECISION: Test two independent samples of 5 each rather than one sample of size 20.

EXAMPLE #3

ACCELERATED TESTING VIA SUDDEN DEATH METHOD

Suppose a Weibull population has a slope of 1.5 and a Characteristic Life of 1,000 Hours. A median typical sample of 5 taken from such a population would yield the following life values:

ORDER STATISTIC NO.	LIFE IN HOURS	
1	268	
2	523	Total Hours
3	783	----------------
4	1101	4,285
5	1610	

On the other hand, if we would conduct a sudden death test on five groups of 8 each, and wait only until the earliest failure in each group, the five earliest failures would have the following median values:

ORDER STATISTIC NO.	LIFE IN HOURS	
1	67	
2	131	Total Hours
3	196	----------------
4	275	1,072
5	403	

CONCLUSION: It can be seen that each earliest failure in 8 is 1/4 of the corresponding order number in the original sample of 5. This all comes about because of the following mathematical formula:

Any Sudden Death Life (i.e., the first of K) is related to the original corresponding ordered life from the population by the formula

$$\text{SUDDEN DEATH LIFE} = \frac{\text{POPULATION LIFE}}{K^{\frac{1}{B}}}$$

Where, K = Sudden Death Group Size and B = Population Slope
(In this case, $K^{1/b} = 8^{(1/1.5)} = 8^{(2/3)} = 4$.)

DECISION: For inexpensive specimens we can accelerate a life test by testing in groups and waiting only until the first failure in each group.

A TYPICAL PROBLEM

QUESTION #1: A good design which has a Reliability of at least 99% to 1,000 hours of operation gains the seller $150, while a design less than 99% reliable to 1,000 hours of operation will cause a loss of $1,800. What confidence level is required for compliance to 1,000 hours with 99% reliability, if the seller wants to gain at least twice as much as he loses on the design in the long run?

QUESTION #2: If the Weibull slope in a life test on the design is 2, and the Test $B_{1\%}$ Life is 1,100 Hours, how large must the test sample size be?

DECISION TO BE MADE: Is the required single sample size too large to be practical, requiring us to try smaller samples in sequence in order to accelerate the whole life testing project into a more reasonable program?

CONCLUSION ARRIVED AT IN THE STUDY OF THIS TYPICAL PROBLEM

It will be found that the required single sample size in this case turns out to be 167 according to the **GOAL CONFIDENCE** Program in DRI's CARS Software Package. Since such a large single sample is not considered practical, we go to testing 6 independent samples of size 5 each. This will yield all the Evidence needed plus a little extra. This is due to the fact that a sample of 167 will yield Evidence proportional to the Square Root of 167, whereas 6 samples of 5 each will yield Evidence proportional to

$$6\sqrt{5} = \sqrt{180} > \sqrt{167} \ .$$

So, it can be seen that it's possible to get by with 30 specimens, i.e., 6 x 5, instead of 167 specimens in the single large sample of size 167.

CONCLUSION

In this bulletin we have demonstrated the fact that it is possible to shorten life tests considerably by testing independent small samples in sequence until the required Evidence Level for Compliance to life goal is reached. Furthermore, the technique of Sudden Testing is very effective when specimens are not very expensive and the earliest failures in a group occur much sooner than average. None of these techniques require any increase in load, and thus there is not the problem of any strange side effects due to overload.