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THE USEFULNESS OF WEIBULL ANALYSIS IN MONTE-CARLO SIMULATIONS FOR PREDICTING FUTURE RANDOM VALUES FOR A VARIABLE UNDER STUDY

INTRODUCTION

Accuracy in statistical predictions on a variable quantity under investigation takes into account the nature of the distribution function of the variable. Among other things, we should take into account the possibility that the variable has a skewed distribution, whose skewness could be either positive or negative. In other words, it wouldn't be a wise choice to use the so-called bell curve (normal distribution), which has no skewness. For this reason alone, in general, it is much more appropriate to use a Weibull distribution as our basis for predictions, especially when dealing with measurements which are always positive numbers, and thus have zero or a positive number as the extreme lower limit for the variable in question.

For example, if we want to predict a bunch of random miles driven by new car customers after so many days of ownership since the delivery date, then we should establish from past experience how the number of miles driven per day of ownership is distributed. For this purpose, it would be wiser to use a Weibull distribution with proper shape and location parameters, since this would eliminate the absurdity of negative values for miles per day. The use of a normal bell curve would be illogical in such a study, since it has an infinite left hand tail, and thus could, at random, yield a negative number of miles per day, which would be a lot of nonsense.

In this bulletin we shall outline a wiser approach to such simulation problems in prediction statistics by resorting to Weibull parameters of past experience on the variable under investigation.

THE BASIC STEPS IN THE MONTE-CARLO SIMULATION PREDICTION PROCESS

In the type of prediction we propose in this bulletin by means of random numbers between 0 and 1, there are three basic steps to be followed. These are listed below:

STEP 1: Look at past data on the variable being studied, and calculate the Weibull parameters. This will yield

(a) The Weibull Slope
(b) The Characteristic Value
(c) The Minimum Possible Value for Measured Variable

STEP 2: Select a random number between 0 and 1. Nowadays this can be done conveniently on any modern computer which has the so-called RND function built into a mathematical program such as BASIC.

STEP 3: Take the random number obtained in **Step 2** and calculate the value of the variable in its Weibull Distribution by using the formula for the B_q level in such a Weibull distribution.

This formula is (with **x** as the variable):

$$x = A + (\theta - A)\left[\left(\ln \frac{1}{1 - RND}\right)^{\frac{1}{b}}\right]$$

Where
A = Minimum (lowest) Possible Value
θ = Characteristic Value (at 63.2% of distribution)
b = Weibull Slope

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A NUMERICAL EXAMPLE OF THE WEIBULL RANDOM PREDICTION TECHNIQUE

STEP 1: Past data on miles driven per day:

Thirty random car owners were observed in a study on a certain model of a passenger vehicle sold at various dealerships engaging in the study. The following list was compiled:

DATA TABLE 1

DAYS OF OWNERSHIP	TOTAL MILES DRIVEN	MILES DRIVEN PER DAY
55	174	3.16
10	61	6.10
75	909	12.12
112	2019	18.03
62	1489	24.02
25	754	30.16
196	7648	39.02
22	1108	50.36
49	3031	61.86
31	2329	75.13
20	1701	85.05
163	16451	100.93
138	1512	10.96
46	697	15.15
81	2126	26.25
122	5213	42.73
105	2015	19.19
59	1075	18.22
70	515	7.36
41	265	6.46
77	861	11.18
158	1541	9.75
26	110	4.23
94	1726	18.36
101	1465	14.50
28	225	8.04
61	371	6.08
105	1602	15.26
39	3012	77.23
57	485	8.51

Next we enter the 30 values from last page for miles driven per day into DRI's LOGWBL program, and we obtain the following Weibull parameters:

Minimum Value	$= A = 0$	miles per day
Weibull Slope	$= b = 1.283$	
Characteristic Value	$= \theta = 29.083$	miles per day

B_{10} = 5.035 miles per day
B_{50} = 21.857 miles per day
B_{90} = 55.708 miles per day

Now, suppose we get a list of 10 random owners of the vehicle model in question, and they give us the following table on the number of days elapsed from delivery date to the failure date of a certain component, say, a water pump:

DATA TABLE 2

CUSTOMER NO.	NO. OF DAYS TO WATER PUMP FAILURE			
1	D_1	=	1765	days
2	D_2	=	1142	days
3	D_3	=	2391	days
4	D_4	=	1325	days
5	D_5	=	992	days
6	D_6	=	1936	days
7	D_7	=	1573	days
8	D_8	=	712	days
9	D_9	=	2506	days
10	D_{10}	=	2157	days

Since we weren't given the odometer readings at these water pump failures, we must assign random mileages (**Step 2**) for the number of days of ownership. In order to get the distribution of miles to water pump failures, we can select a random number between 0 and 1, and then calculate the miles per day (**x**) corresponding to this random quantile in the Weibull distribution for miles driven per day. Multiplying this random miles per day by the number of days to the water pump failure for any customer, we obtain a predicted random number of miles to a water pump failure for that customer. Then we can make a Weibull plot for miles to a water pump failure, and thus get a basis for predicting water pump life in miles.

For customer No. i we select RND_i,a random number between 0 and 1. Then we calculate his or her mileage to a water pump failure by the formula (**Step 3**):

$$M_i = D_i \cdot \theta \cdot \left[\left(\ln \frac{1}{1 - RND_i} \right)^{\frac{1}{b}} \right]$$

Where, θ = 29.083 miles/day (Characteristic Value)
and, b = 1.283 (Weibull Slope)

The following GW-BASIC program ("DAYMIPRG") would handle this program:

```
10 PRINT "DAYMIPRG"
20 INPUT "CHARACTERISTIC VALUE (MILES) PER DAY =";T
30 INPUT "WEIBULL SLOPE FOR (MILES/DAY) =";B
40 INPUT "TIMES (DAYS) TO FAILURE =";D
50 RANDOMIZE TIMER
60 F = RND
70 M = D*T*((LOG(1/(1-F)))^(1/B))
80 PRINT "MILES TO FAILURE =";INT(M+.5)
90 LPRINT "MILES TO FAILURE =";INT(M+.5)
100 PRINT
110 LPRINT
120 INPUT "ANOTHER CUSTOMER (Y/N)";Y$
130 IF Y$="Y" OR Y$="y" THEN 40
140 PRINT "END OF DATA SET"
150 LPRINT "END OF DATA SET"
```

When we enter into this program the days to failure for the list of 10 customers in Data Table 2, we get the following list of 10 random miles to failure:

TABLE 3: TEN RANDOM MILES TO WATER PUMP FAILURE

Miles to Failure	=	31653
Miles to Failure	=	9973
Miles to Failure	=	83742
Miles to Failure	=	32708
Miles to Failure	=	18300
Miles to Failure	=	33135
Miles to Failure	=	20827
Miles to Failure	=	18012
Miles to Failure	=	82636
Miles to Failure	=	23442

Finally, we enter these 10 random miles to failure into DRI's "LOGWBL" Program, and obtain the following Weibull parameters for water pump life in miles:

Weibull Slope	=	1.631
Characteristic Life	=	39891 miles
B_{10} Life	=	10035 miles
B_{50} Life	=	31863 miles
B_{90} Life	=	66521 miles

CONCLUSION

The convenience of using the Weibull distribution in predicting the water pump life in miles has been illustrated by a situation in which we were only given the days to water pump failures. What was necessary was to have past data on the Weibull distribution of miles driven per day. From the Weibull parameters on such past data we derived random mileages to water pump failures, and fitted a Weibull distribution to the random mileages so generated.