STATISTICAL BULLETIN
Reliability & Variation Research

LEONARD G. JOHNSON
EDITOR

DETROIT RESEARCH INSTITUTE
P.O. Box 36504 • Grosse Pointe, MI 48236 • (313) 886-8435

WANG H. YEE
DIRECTOR

Volume 26
Bulletin 7

January, 1997
Page 1

# USING WEIBULL PAPER TO GRAPHICALLY REPRESENT GUARANTEED MAXIMUM FAILURE RATES FOR ANY DESIRED LEVEL OF ODDS

## Introduction

There is the story of a product purchaser who was shown a Weibull plot for a life test on the product he was going to purchase. The customer wanted the product to run 1,000 hours at normal operating conditions with no more than 1 item in 10 failing in the 1,000 hours. The Weibull median rank plot was for a sample of 20 items run to failure, and it showed 5% failed in 1,000 hours. The customer wanted to know the odds in favor of not exceeding 10% failed in 1,000 hours. The manufacturer told the customer to come into the statistical analysis section of the testing lab to discuss his question. The chief statistician, whose name was Gus, was introduced to the customer. "What's your problem?", asked Gus. The customer told Gus that he was concerned with the question of deriving realistic confidence bounds for median rank Weibull plots. "For example", he said, "I was just shown a Weibull plot on 20 items tested to failure which showed 5% failed at 1,000 hours. What I want to know is the **Odds** in favor of not having more than 10% failed in the 1,000 hours." "Well", replies Gus, "What you need is a graph picturing how the **Maximum Fraction Failed** is related to the **Odds**. This can be figured out by using the practical and reasonable theory of **Semi-Parametric Ranks.**"

This bulletin discusses the mathematics involved in this question, and how it can be graphically pictured as a straight line with **Odds** as abscissa and the **Maximum Fraction Failed** as ordinate. Interestingly enough, this new type of **Linear Prediction Line** is constructed on Weibull paper, which agrees with **Semi-Parametric** ranking theory.

## SEMI-PARAMETRIC STATISTICAL RELIABILITY MATHEMATICS

We begin with the basic **Universal Law of Odds** developed by Detroit Research Institute back in 1986, which states that

$$\text{MEDIAN ODDS} = (\text{ENTROPY RATIO})^{(\text{Median Odds Exponent})}, \qquad (1)$$

where the median odds are what we get from Semi-Parametric ranks, and are thus considered the most reasonable predictors, and the **Entropy Ratio** is defined to be

$$\frac{\text{Natural Logarithm of Reliability with Desired Odds}}{\text{Natural Logarithm of Reliability with Even Odds}}$$

The Reliability with Even Odds at any test life x (like 1,000 hours) is obtained by taking the fraction failed on a median rank Weibull plot at the test life, and subtracting that fraction failed from unity. Thus,

$$\text{Reliability with Even Odds} = R_{.50}(x) = 1 - \text{Fraction Failed at } x$$

(on median rank Weibull plot)

The MEDIAN ODDS EXPONENT $= \dfrac{\sqrt{.5N(1+Q_{.50})}}{.55}$, where

N = Sample Size at $x_0$

Q = Fraction Failed at $x_0$ (on median rank Weibull plot)

Equation (1) can be solved for the **Entropy Ratio** as follows:

$$\text{ENTROPY RATIO} = \frac{\ln R_c(x_o)}{\ln R_{.50}(x_o)} = (\text{MEDIAN ODDS})^{\frac{.55}{\sqrt{.5N(1+Q_{.50})}}}$$

Thus, we can write the following equation:

$$\ln\ln\frac{1}{R_c(x_o)} = \frac{.55}{\sqrt{.5N(1+Q_{.50})}}\ln(\text{Median Odds}) + \ln\ln\frac{1}{R_{.50}(x_o)} \qquad (2)$$

---

By examining equation (2) we see that it is analogous to a Weibull line with the following modifications:

Instead of having **Life x** as abscissa, we have (**Median Odds**) as abscissa.

The slope of the line on Weibull paper is $\dfrac{.55}{\sqrt{.5N(1+Q_{.50})}}$ .

So, we have at 1:1 Odds (50% confidence):

Fraction Failed = $Q_{.50}$ (From the Life Test Weibull plot at $x_0$)

At 100:1 Odds [Confidence = 100/101 = 0.990099] :

$$Q_c = 1 - (1 - Q_{.50})^{100^{\frac{.55}{\sqrt{.5N(1+Q_{.50})}}}}$$

This quantity $Q_c$ is the promised **Maximum Fraction Failed** at $x_0$ with Odds of 100:1.

## APPLYING THE THEORY TO THE CUSTOMER'S EXAMPLE

In the customer's example, we have

$x_0$ = 1,000 Hours
$Q_{.50}$ = .05 (5% failed on the Weibull plot)
N = 20 (Sample Size)

So, for 100:1 Odds (i.e., for Confidence = 100/101 = 0.990099):

$$Q_c = 1-(1-.05)^{100^{\frac{.55}{\sqrt{.5(20)(1+.05)}}}} = 1-.95^{100^{(.169734)}} = 1-.95^{(2.18508)}$$

$$Q_c = 1-0.89397 = 0.10603$$

Thus, we can promise with 100:1 Odds that not more than 10.6% will fail in 1,000 hours.

For exactly 10% as the Maximum Fraction Failed, as the customer asked, the Odds would be

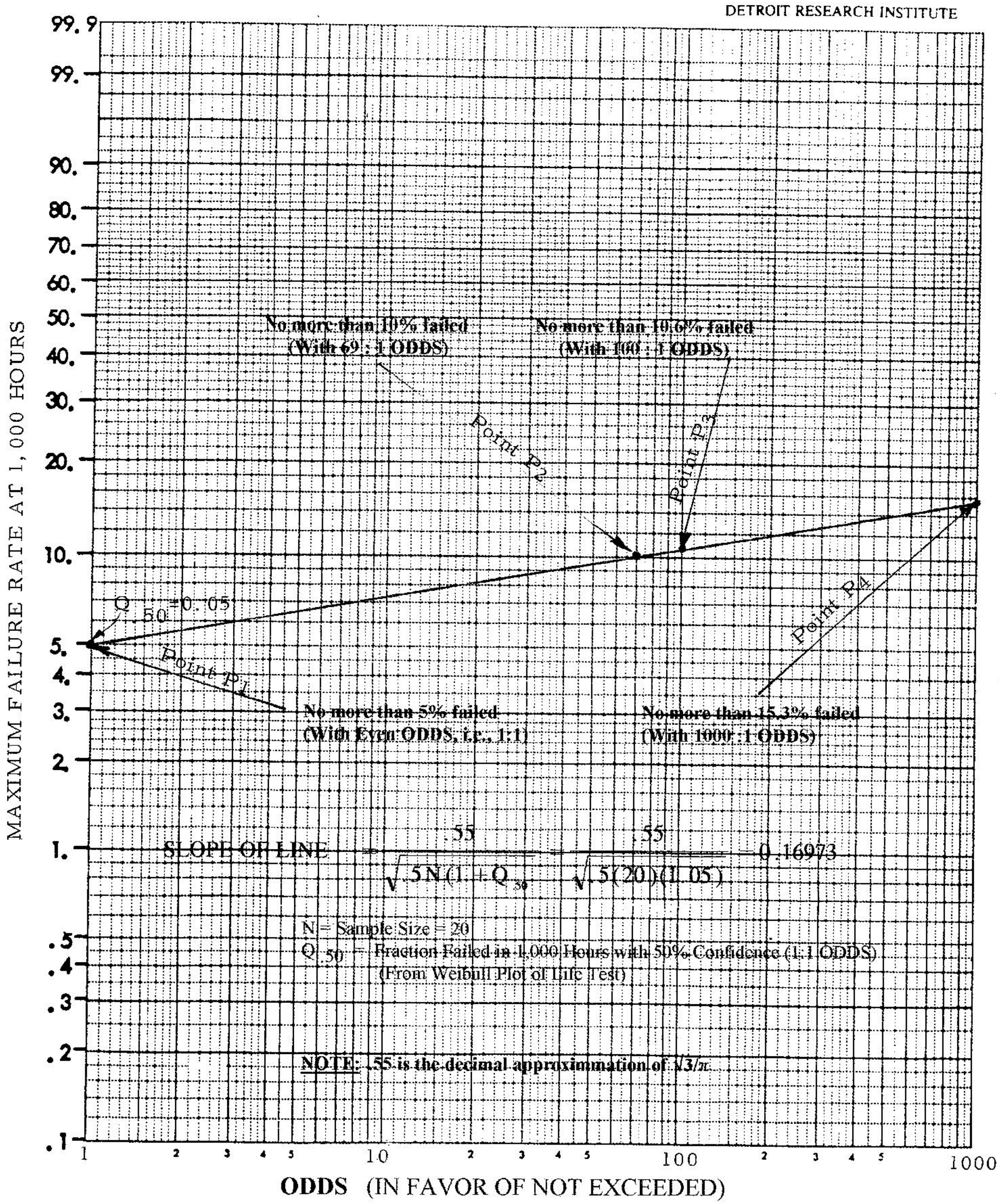
$$\text{ODDS} = \left(\frac{\ln .90}{\ln .95}\right)^{\frac{\sqrt{.5(20)(1+.05)}}{.55}} = (2.05408)^{5.89158} = 69.47$$

Thus, Gus told the customer that the **Odds** in favor of not exceeding 10% failed in 1,000 hours is 69.47 to 1, which comes out to a **Confidence** of 69.47/70.47 = 0.9858.

The linear plot on Weibull paper for this example is shown in Figure 1.

FIGURE 1

**PLOT OF PERCENT FAILED IN 1,000 HOURS VS. ODDS OF NOT EXCEEDED**



## CONCLUSION

As has been shown in our discussion of the question posed by the customer in our story, the Weibull type of mathematical relation has turned out to have another handy application in predicting reliability and failure rate levels with specified odds in favor of not exceeding any desired worst tolerable failure percentage. The linear graph on Weibull paper makes it so easy to graphically read the maximum failure rate for any desired level of odds. Once again, the Weibull function has come to our rescue as far as graphical presentations in reliability are concerned.

# APPENDIX

As additional useful information regarding the type of question discussed in this bulletin, we present two more items of interest.
These are:

(1) The convenience of a linear plot on **log-log Entropy** graph paper for reading off paired values of **Odds** and corresponding **Maximum Entropies** which can be promised.

(2) A computer program developed by Detroit Research Institute to automatically calculate (at any life):
   (a) The Maximum Fraction Failed for specified **Odds Level.**
   (b) The **Odds** corresponding to any promised Maximum Fraction Failed.

The computer printout for the example in this bulletin is given on Page 8.

To construct the **Entropy** linear plot (Figure 2) for our example, we make the following modifications:

For Point $P_1$: 5% failed is made into: Entropy = -ln.95 = .0513
For Point $P_2$: 10% failed is made into: Entropy = -ln.90 = .1054
For Point $P_3$: 10.6% failed is made into: Entropy = -ln.894 = .1120
For Point $P_4$: 15.3% failed is made into: Entropy = -ln.847 = .1660

Then the four points $P_1$, $P_2$, $P_3$, and $P_4$ are given Entropy ordinates of .0513, 1054, .1120, and .1660, respectively. These points will then line up perfectly on **log-log Entropy** graph paper, as shown in Figure 2 on Page 9.

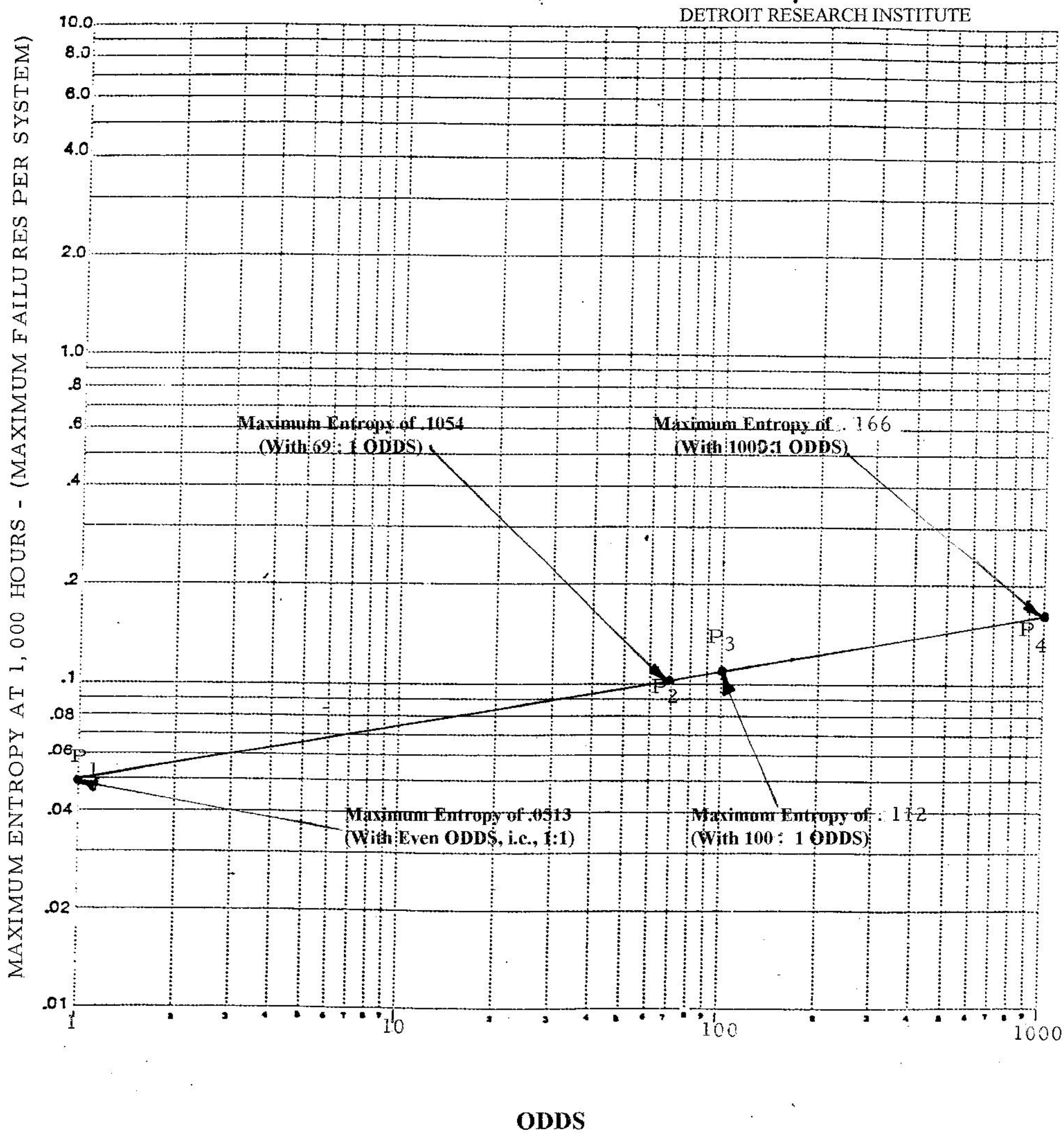
<u>**NOTE:**</u> The definition of **Entropy** is **"FAILURES PER SYSTEM"** for any **Repairable System.**

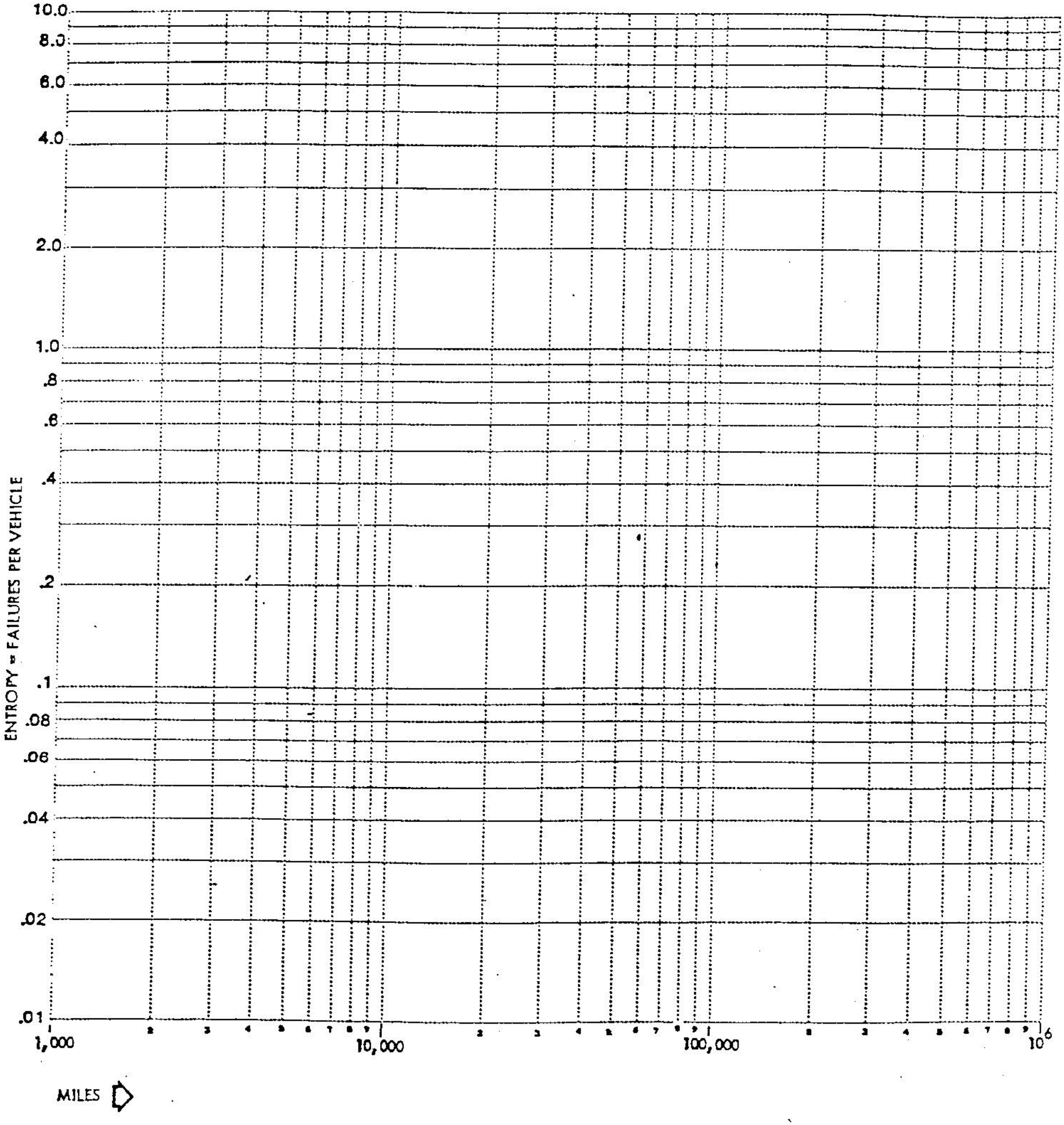
PROGRAM FOR CALCULATING MAXIMUM FRACTION FAILED AND ODDS (MFFODDS)

```
ENTER SPECIFIED MAXIMUM FRACTION FAILED ALLOWED =? .1
ENTER MEDIAN FRACTION FAILED (FROM WEIBULL PLOT) =? .05
SAMPLE SIZE = 20

REQUIRED ODDS AT SPECIFIED MAX. FRACTION FAILED = 69.47229
CORRESPONDING CONFIDENCE = .9858101

WHAT OTHER ODDS DESIRED = 100

ODDS DESIRED =  100
CORRESPONDING CONFIDENCE = .990099
MAXIMUM FRACTION FAILED = .1060272

ANOTHER ODDS DESIRED - Y/N? y

WHAT OTHER ODDS DESIRED = 1000

ODDS DESIRED =  1000
CORRESPONDING CONFIDENCE = .999001
MAXIMUM FRACTION FAILED = .1526799

ANOTHER ODDS DESIRED - Y/N?N
Ok
```

FIGURE 2

## ENTROPY PLOT FOR MAXIMUM FAILURES PER SYSTEM IN 1,000 HOURS VERSUS THE ODDS SHOWN ON THE ABSCISSA SCALE






Leonard G. Johnson / Henry L. Pouch