STATISTICAL BULLETIN
Reliability & Variation Research

LEONARD G. JOHNSON
EDITOR

DETROIT RESEARCH INSTITUTE
P.O. Box 36504 • Grosse Pointe, MI 48236 • (313) 886-8435

WANG H. YEE
DIRECTOR

Volume 24
Bulletin 2

May, 1994
Page 1

THE WEIBULL ENTROPY AVERAGE NO-FAIL THEOREM AND ITS APPLICATIONS TO LIFE TESTS IN WHICH NONE OF THE TESTED ITEMS HAVE FAILED AS YET

INTRODUCTION

A common problem encountered in life testing is one in which none of the tested items has failed, in spite of the fact that the test items have been run for such a long time that it is no longer desirable to run them any longer due to the fact that test equipment and testing time are limited. As a consequence of being in such a situation, we are faced with the problem of coming up with some sort of reasonable reliability estimate for the product which has been tested, even though none of them has failed. In this bulletin we discuss what we call **The Weibull Entropy Average No-Fail Theorem**. By means of this theorem we can make a conservative prediction of the product's reliability to any desired target, provided that we have a reasonable estimate of the **Weibull Slope** for the product in question. A computer program has been devised in order to make the mathematical procedure easy to employ in any such situation where we need a quick estimate of the product's reliability rating to any life target, and thus obtaining what information is needed in accepting or rejecting the product as far as durability is concerned.

ESTIMATING CHARACTERISTIC LIFE FROM A SET OF NON-FAILURES

It has been our practice in the past to take a sample of non-failures with life values $x_1, x_2, x_3, \ldots\ldots x_N$ in a Weibull Distribution of slope b and come up with a **very conservative** estimate of the **Characteristic Life** of the population by using the formula

$$\hat{\theta}_{cons.} = (2x_1{}^b + x_2{}^b + x_3{}^b + \;\ldots\ldots + x_N{}^b)^{(1/b)}$$

$$(x_1 = \text{shortest life})$$

As we stated above,, this is a **very conservative estimate** of **Characteristic Life**. What we propose in this bulletin is a much more reasonable estimate (but still conservative) for the **Characteristic Life**. This more reasonable formula looks as follows:

$$\hat{\theta}_{ave.} = \left\{[(x_1 + x_2 + \cdots x_N)/N]^b + x_1{}^b + x_2{}^b + x_3{}^b + \ldots x_N{}^b\right\}^{(1/b)}$$

If all the x_i's are equal to x_{test} then the above formula becomes

$$\hat{\theta}_{ave.} = (N + 1)^{1/b}(x_{test})$$

This more reasonable estimate of **Characteristic Life** we call **The Weibull Entropy Average No-Fail Theorem**. On the next page we list the Computer Program in BASIC for automating this prediction procedure for tests having no failures. We close with a couple of actual examples of such testing situations.

```
11200 COLOR 15,1:CLS
11210 LOCATE 3,30
11220 PRINT"AVERAGE NO-FAIL PROGRAM "
11225 DIM X(100)
11230 PRINT
11240 A1=0:S=0
11245 INPUT "QUANTILE LEVEL UNDER STUDY IN TEST PROGRAM =";Q
11250 INPUT"                 WEIBULL SLOPE=";B
11260 INPUT"                 NO.OF NON-FAILURES=";K
11270 PRINT:PRINT
11290 FOR I=1 TO K
11300 PRINT "                 LIFE NO. ";I
11310 INPUT "                 VALUE = ";X(I)
11315 A=X(I):A1=A1+X(I):A2=A1/I
11320 T=X(I)^B
11330 S=S+T
11340 NEXT I
11350 W=(A2^B+S)^(1/B)
11360 CLS:PRINT:PRINT
11370 PRINT TAB(30)"CHAR.LIFE=";W
11380 Z=W*((LOG(1/.9))^(1/B))
11385 Z1=W*((LOG(1/(1-Q)))^(1/B))
11390 PRINT TAB(30)"B-10 LIFE=";Z
11400 PRINT TAB(30)"B("Q;") LIFE=";Z1
11410 INPUT"           FIRST TARGET LIFE=";Y
11420 R=EXP(-(Y/W)^B):PRINT:PRINT
11430 PRINT"           RELIABILITY TO FIRST TARGET - (WITH 50 % CONF.) =";R
11440 PRINT:PRINT
11450 INPUT "           OTHER CONF. LEVEL DESIRED - (DECIMAL VALUE) = ";C
11460 INPUT "           NO. UNFAILED BEFORE FIRST TARGET = ";N1
11470 N2=K+1-N1
11480 PRINT "           SAMPLE SIZE AT FIRST TARGET =";N2
11490 G=(C/(1-C))^(.55/SQR(N2*(1-.5*R)))
11500 R1 = R^G
11510 PRINT "           RELIABILITY TO FIRST TARGET - (WITH";100*C;"% CONF.) =";R1
11520 PRINT
11530 INPUT "           SECOND TARGET LIFE=";V
11540 P=EXP(-(V/W)^B)
11550 PRINT "           RELIABILITY TO SECOND TARGET - (WITH 50 % CONF.) =";P
11560 INPUT "           NO. UNFAILED BEFORE SECOND TARGET = ";N3
11570 N4=K+1-N3
11580 PRINT "           SAMPLE SIZE AT SECOND TARGET =";N4
11590 H=(C/(1-C))^(.55/SQR(N4*(1-.5*P)))
11600 P1=P^H
11610 PRINT "           RELIABILITY TO SECOND TARGET - (WITH";100*C;"% CONF.) =";P
1
11620 PRINT:PRINT
11630 INPUT "           ANOTHER CONF. LEVEL ..... (Y/N - TYPE N IF NOT DESIRED)";Y
$
11640 IF Y$="Y" OR Y$="y" THEN 11440:PRINT:PRINT
11650 INPUT "           ANOTHER RUN ..... Y/N";Y$
11660 IF Y$="Y" OR Y$="y" THEN 11230
11670 END
```

EXAMPLE # 1

```
                    AVERAGE NO-FAIL PROGRAM

QUANTILE LEVEL UNDER STUDY IN TEST PROGRAM =? .632
         WEIBULL SLOPE=? 2
         NO.OF NON-FAILURES=? 3


         LIFE NO.  1
         VALUE = ? 1000
         LIFE NO.  2
         VALUE = ? 1100
         LIFE NO.  3
         VALUE = ? 1500


                    CHAR.LIFE= 2428.991
                    B-10 LIFE= 788.433
                    B( .632 ) LIFE= 2428.593
      FIRST TARGET LIFE=? 500


      RELIABILITY TO FIRST TARGET - (WITH 50 % CONF.) = .9585123


      OTHER CONF. LEVEL DESIRED - (DECIMAL VALUE) = ? .9
      NO. UNFAILED BEFORE FIRST TARGET = ? 0
      SAMPLE SIZE AT FIRST TARGET = 4
      RELIABILITY TO FIRST TARGET - (WITH 90 % CONF.) = .9067494

      SECOND TARGET LIFE=? 800
      RELIABILITY TO SECOND TARGET - (WITH 50 % CONF.) = .8972017
      NO. UNFAILED BEFORE SECOND TARGET = ? 0
      SAMPLE SIZE AT SECOND TARGET = 4
      RELIABILITY TO SECOND TARGET - (WITH 90 % CONF.) = .7829008


      ANOTHER CONF. LEVEL ..... (Y/N - TYPE N IF NOT DESIRED)?
```

EXAMPLE # 2
AVERAGE NO-FAIL PROGRAM

```
QUANTILE LEVEL UNDER STUDY IN TEST PROGRAM =? .01
            WEIBULL SLOPE=? 3
            NO.OF NON-FAILURES=? 7


            LIFE NO.  1
            VALUE = ? 10
            LIFE NO.  2
            VALUE = ? 12
            LIFE NO.  3
            VALUE = ? 20
            LIFE NO.  4
            VALUE = ? 30
            LIFE NO.  5
            VALUE = ? 45
            LIFE NO.  6
            VALUE = ? 70
            LIFE NO.  7
            VALUE = ? 90


                            CHAR.LIFE= 108.0887
                            B-10 LIFE= 51.05121
                            B( .01 ) LIFE= 23.32589
        FIRST TARGET LIFE=? 5


        RELIABILITY TO FIRST TARGET - (WITH 50 % CONF.) = .999901


        OTHER CONF. LEVEL DESIRED - (DECIMAL VALUE) = ? .95
        NO. UNFAILED BEFORE FIRST TARGET = ? 0
        SAMPLE SIZE AT FIRST TARGET = 8
        RELIABILITY TO FIRST TARGET - (WITH 95 % CONF.) = .9997776

        SECOND TARGET LIFE=? 15
        RELIABILITY TO SECOND TARGET - (WITH 50 % CONF.) = .9973309
        NO. UNFAILED BEFORE SECOND TARGET = ? 2
        SAMPLE SIZE AT SECOND TARGET = 6
        RELIABILITY TO SECOND TARGET - (WITH 95 % CONF.) = .9932238


        ANOTHER CONF. LEVEL ..... (Y/N - TYPE N IF NOT DESIRED)?
```

CONCLUSION

We have presented **The Weibull Entropy Average No-Fail Theorem**, and including the BASIC Computer Program for predicting the value of the Characteristic Life, as well as any other desired Quantile Level in the conservatively estimated Weibull Population of assumed slope **b**, together with the Reliability to Any Desired Life Target, with 50% confidence and other Desired Level of Confidence for the Reliability to the Specified Target. This automatic computing procedure is indeed one of handiest methods of making sense out of those annoying situations in which we are unable to schedule a long enough running time for test specimens of such long life that they do not fail in the time we allow in testing them.