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Reliability & Variation Research

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SIGNIFICANCE AND CONFIDENCE FACTORS IN LIFE TESTING AND HOW THEY RELATE TO A MANUFACTURER'S PROFITABILITY

INTRODUCTION

Once there was engineer and designer of mechanical systems. His name was Vic, and he was famous for the ingenuity he had shown for many years in making designs reliable in service when they were put to the use they were intended for. All his working partners respected this man's complete understanding of all the parameters which had to be taken into consideration before any design could ever be accepted as sufficiently reliable to both satisfy buyers and profitable to the manufacturer. Vic, the very ingenious craftsman, was always preaching his reliability philosophy to his bosses and coworkers. "What are the main points in your approach to the problem of product reliability?" asked one of his working partners one day in the test lab where the company's product testing was being done. "Well", answered Vic, "Have you ever heard about the Concepts of Significance and Confidence?". "Oh, I've heard such terms tossed around here in our laboratory, but what they really represent is not entirely clear to me", replied the partner. Then Vic suggested that a special training session should be set up in which he could explain to all the lab workers what kind of an approach to reliability testing he had developed during his many years of experience with proposed mechanical designs.

Vic's First Lecture --- The Concept of Significance

What is meant by the significance of a life safety factor for a design intended to endure a specified or required length of life in service? This required length of life we shall call the **Goal Life** or **Baseline Life.** When a life test is conducted on a sample of items of a certain design we have become accustomed to constructing a **Weibull Plot** of the sample life values on **Weibull Probability Paper** to give the best predicted values of such things as the B-10 Life (where 10% is predicted to be failed), or the B-50 Life (called the **Median Life**, where 50% is predicted to be failed), or B-90 Life (where we predict that 90% of the items will be failed). We can talk about the **B-10 Goal**, or the **B-50 Goal,** or the **B-90 Goal**, or any other **B-Q Goal** (where **Q** is called **Quantile Level** or **Fraction Failed**). If, for example, a Weibull Plot shows a test value at the B-10 Level to be at 1,500 hours, while the B-10 Goal Life is only 1,000 hours, this means that there is an apparent improvement ratio 1500/1000 or 1.5 with respect to the B-10 Goal. The **significance** of this safety factor is the **probability** that this apparent amount of improvement is an indication of **any improvement** of the design over and above the required B-10 goal of 1,000 hours. In other words, **Significance** is the probability that the **True Safety Factor** of the design is **at least unity.** The observed ratio 1500/1000 has only a probability **(confidence)** of 50% when we use **Median Rank** plotting in our **Weibull Analysis**. Thus, **Significance** is only the **upper point** on the y-axis of a **Confidence Map** (or **Confidence Interpolation Diagram**) for the product's ability to exceed the Required Goal Life. In other words, **Significance** is the confidence of being **at least as good as the Goal Life**. On the other hand, the **Observed Life Improvement Ratio**, such as 1500/1000 has only a 50% confidence attached to it, and thus is the **lower point** (on the x-axis) of the **CONFIDENCE INTERPOLATION DIAGRAM**, as shown in Figure 1 on Page 3:

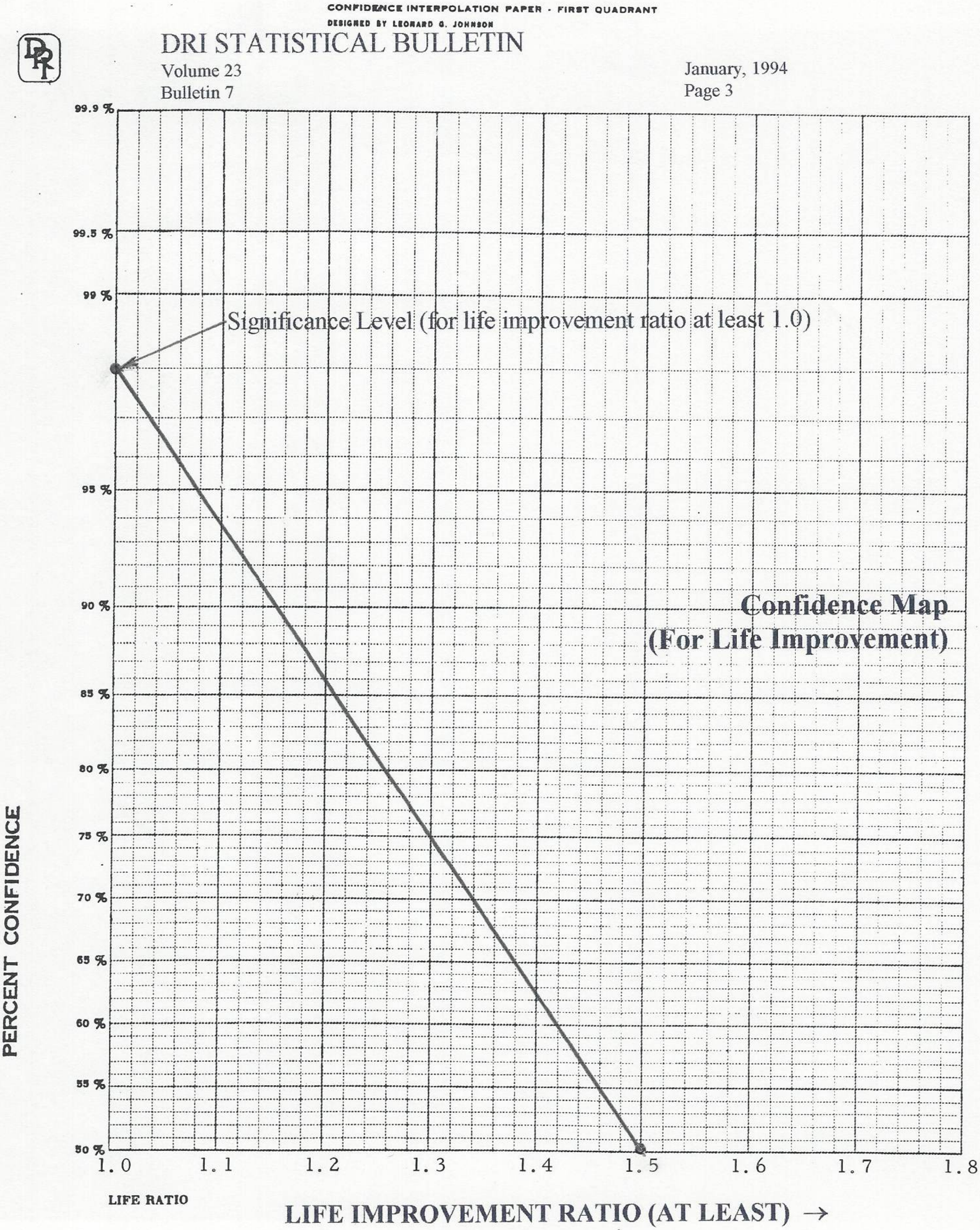

CONFIDENCE INTERPOLATION PAPER - FIRST QUADRANT
DESIGNED BY LEONARD G. JOHNSON
Significance Level (for life improvement ratio at least 1.0)
Confidence Map
(For Life Improvement)
PERCENT CONFIDENCE
99.9 %
99.5 %
99 %
95 %
90 %
85 %
80 %
75 %
70 %
65 %
60 %
55 %
50 %
1.0
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
LIFE RATIO
LIFE IMPROVEMENT RATIO (AT LEAST) →

Figure 1

Vic's Second Lecture : Every Confidence Number Must have A Corresponding Hypothesis

It's meaningless to talk about **confidence** unless we know what is the **hypothesis** corresponding to the confidence number. For example, when we talked about **significance** we were referring to the hypothesis that the population from which the test sample was taken has a life **at least** equal to the **Goal Life**. This is the same as the confidence that the design's life is at least equal to the **Required Goal Life**, and this confidence is called the **Significance Level** of the observed test life. On the other hand, if we consider the **hypothesis** that the life safety factor ratio is at least **Test Life/Goal Life** (1500/1000 in the example), we only have 50% confidence for the hypothesis (due to Median Rank plotting). For any intermediate hypothesis about the improvement ratio being between **unity** and the **Observed Ratio** (1.5 in the example), such as 1.25, we refer to the **Confidence Map**, or Confidence Interpolation Diagram, and from it read the corresponding confidence between the **Significance Level** and 50%on the line joining the **upper point (Significance)** on the y-axis and the **lowest point** to the right on **Life Improvement Ratio Scale** along the x-axis, where there is only 50% confidence.

Vic's Third Lecture: Determining The Needed Significance Of A Life Test

Question: How much confidence **(significance)** do we need from a life test in order to realize a **Profitability Ratio K**? This means that **Gains** from a good population, which complies with the required goal life, will be **at least** k times as large as the **losses** suffered from a bad (non-complying) population.

The Answer to the Above Question on the Required Significance

Let C = Confidence of Compliance, i.e., The Probability That Test Life ≥ Goal Life .

Then, 1 - C = Probability That Test Life < Goal Life

And C/(1 - C) = **Odds in Favor of Compliance**

Suppose G = Dollars Gained When in Compliance
and L = Dollars Lost When Not Complying

In order to make the **Expected Value** of **Gains**, i.e., CG, to be at least K time as large as the **Expected Value** of **Losses**, L(1 - C),
We must satisfy the relation

$$CG \geq KL(1 - C)$$

or $$C/(1 - C) \geq KL/G$$

or **Required Odds** ≥ KL/G .

Then, the **Required Significance Level** is the value of C such that

$$C \geq KL/(G + KL)$$

Vic's Fourth Lecture --- The Required Test Sample Size

The odds required in order to realize a **Profitability Safety Factor K** [i.e., K times as many dollars gained (G) due to compliance to a **Life Goal** than will be lost (L) from non-compliance] are given by the formula

$$\text{Odds Required} = KL/G \ .$$

This implies a Confidence C (of compliance) given by

$$C = \text{Odds}/(1 + \text{Odds}) = (KL/G)/(1 + KL/G) = 1/(1 + G/KL)$$

Let ρ = Life Safety Factor = Test Life/Goal Life

Let b = Weibull Slope of Test Data

Let Q = Quantile level under consideration
(If we are studying B-10 compliance, then Q = .1)

The Sample Size required to realize the required **Significance Level** for the life test is given by the formula

$$N = \frac{6}{\pi^2 b^2 (1+Q)} \left(\frac{\ln(KL/G)}{\ln \rho} \right)^2$$

$$= \frac{6}{1+Q} \left(\frac{\ln(KL/G)}{\pi b \ln \rho} \right)^2$$

Vic's Fifth Lecture: Use of Small Trial Tests When We Have No Idea What The Life Safety Factor Is In Advance

In case we have no advance idea what the Life Safety Factor from a life test might be, it is best to test just a *small sample* (say, 5 items) to failure, and then calculate the confidence (Significance Level) by calculating Odds in Favor of at least a Unit Ratio for Population Life/Goal Life by obtaining

$\rho_1 = (\text{Test Life}_1/\text{Goal Life}_1)$ and then calculating the ODDS by the formula

$$\Omega_1 = \text{ODDS (of Significance from Test \#1)}$$

$$= \rho_1^{\pi b_1 \sqrt{\frac{N_1(1+Q)}{6}}}$$

where N_1 = A Small Initial Sample Size (such as 5)
b_1 = Test Sample Weibull Slope

If Ω_1 does not come out to be at least KL/G, then we simply test another *small sample* (say of size N_2) and come up with another **Second Odds** in favor of compliance, as given by the formula

$$\Omega_2 = \text{ODDS (of Significance from Test \#2)}$$

$$= \rho_2^{\pi b_2 \sqrt{\frac{N(1+Q)}{6}}} \quad \text{where } (\rho_2 = \text{Test Life}_2/\text{Goal Life})$$

We then *multiply* $\Omega_1\Omega_2$ to obtain the **Resultant Odds $\hat{\Omega}$.** If $\hat{\Omega} > KL/G$, then we can accept the design as being sufficient to yield the desired **Profitability Ratio**. We can continue this type of **Sequential Testing** until we obtain sufficient **Resultant Odds** > KL/G (by *multiplying* successive test odds).

Vic's Sixth Lecture : An Actual Example

Suppose that a certain design must have a B-10 life of at least 1,000 hours. The profit from selling a complying product is $500,000, while a non-complying product loses $6,000,000. If we want to gain **twice** as much from all complying lots as we would lose from non-complying ones, then the **Required Odds** would be

$$\frac{2(6{,}000{,}000)}{500{,}000} = 24 \text{ to } 1 \quad .$$

Now, suppose that we test a sample of 5 items and obtain the following set of times to failure:

1,270 Hrs., 1,680 Hrs., 2,205 Hrs., 2,618 Hrs., and 3,210 Hrs.

These five lives are plotted on Weibull paper (see Figure 1) to yield a Weibull Slope b = 2.83 and a B-10 Life = 1,121 Hrs.

By using the **Universal Law of Odds**, which says that

$$\mathit{Odds} = (\text{Life Safety Factor})^{\frac{b_1\sqrt{.5(1+Q)N_1}}{.55}}$$

(Note: $.55 = \sqrt{3}/\pi$) $b_1 = 2.83$; $N_1 = 5$

We calculate

$$\mathit{Odds} = (1121 \;/ 1000)^{\frac{2.83\sqrt{.5(1+.1)5}}{.55}} = (1.121)^{8.532} = 2.65$$

This odds (2.65) is not up to the required 24:1 . Thus, further test samples are needed.

Suppose that we next test a set of 8 specimens, and we find the following lives (in hours):

1,250, 1,690, 2,305, 2,865, 3,351, 3,760, 4,301, and 4,875 .

Plotting these on Weibull paper we get a Weibull Slope b = 2.31 and a B-10 Life = 1,315 Hrs. (See Figure 2)

Now, from these parameters, the Odds become

$$Odds = \left(\frac{1315}{1000}\right)^{\frac{2.31\sqrt{.5(1+.1)8}}{.55}} = (1.315)^{8.81} = 11.16 \quad .$$

(Note: $b_2 = 2.31$; $N_2 = 8$)

Thus, after these two test runs, the **Resultant Odds** are (2.65)(11.16 = 29.57, which exceeds the required 24:1, and allows us to accept the product as capable of given the desired profitability.

Conclusion

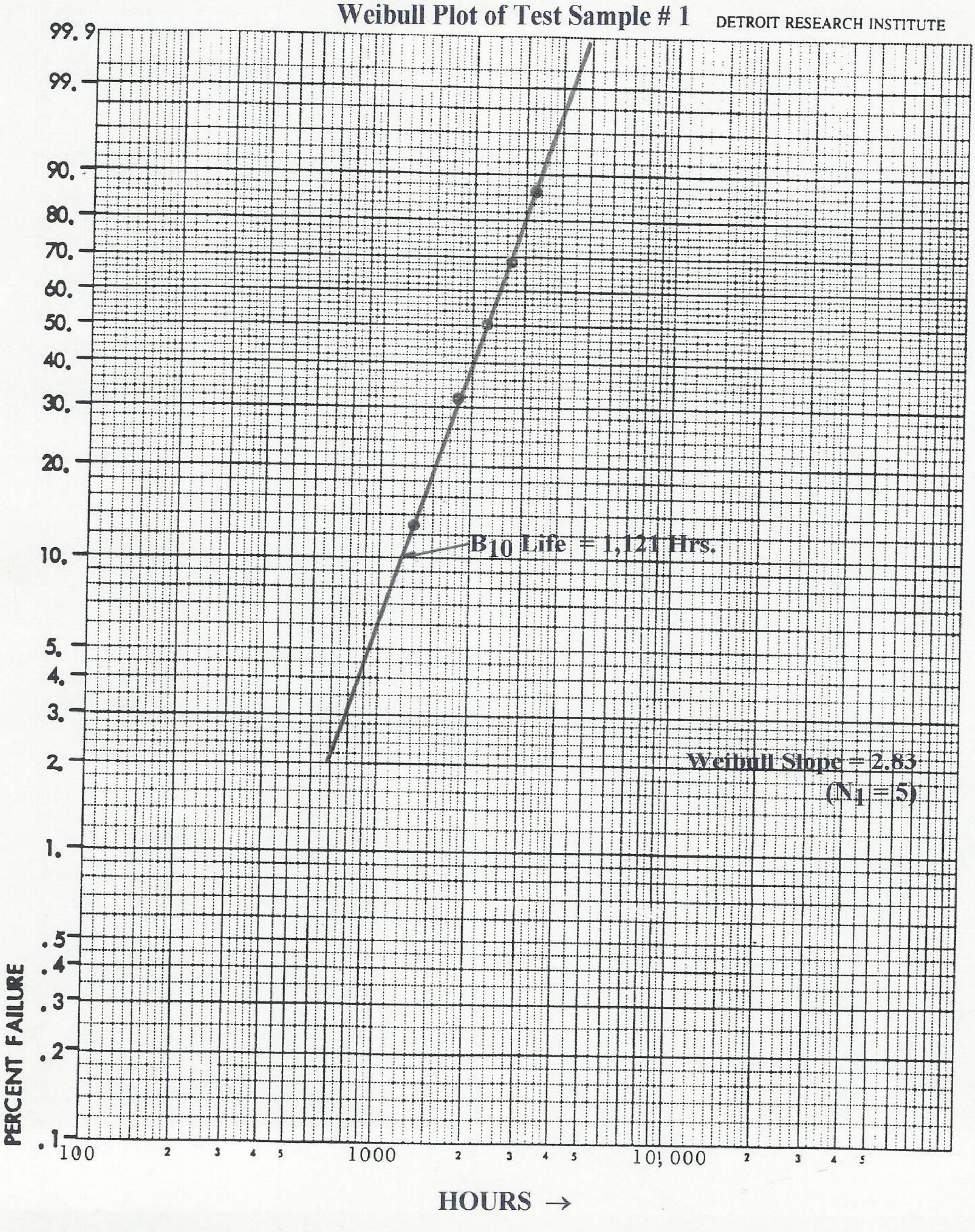
From the example which Vic worked out, we saw that in that particular problem we tested a total of 13 specimens,, i.e., 5 in Test #1 and 8 in Test #2. The weighted average of the two life safety factors is [5(1.121) + 8(1.315)]/13 = 1.240 . Furthermore, the weighted average of Weibull slopes is [5(2.83) + 8(2.31)]/13 = 2.51 .

According to Vic's fourth lecture (on sample size), the sample size required for a 1.24 ratio of (Test Life/Goal Life), with a Weibull slope 2.51, is given by the formula

$$N = \left(\frac{6}{1+.1}\right)\left(\frac{\ln(24)}{\pi(2.51)\ln(1.24)}\right)^2 = 19.15 \text{ or } 20 \text{ to next integer}$$

Thus, with tests in sequence we get by with 13 specimens, while a single test with the weighted life ratio safety factor and weighted Weibull slope would have to be of size 20. This illustrates the specimen saving power of sequential testing.

All in all, it can be seen that Vic, the testing expert, has developed a neat and straightforward approach to the design and analysis of life testing experiments.


Weibull Plot of Test Sample # 1
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99. 9
99.
90.
80.
70.
60.
50.
40.
30.
20.
10.
5.
4.
3.
2.
1.
.5
.4
.3
.2
.1
PERCENT FAILURE
100
2
3
4
5
1000
2
3
4
5
10,000
2
3
4
5
HOURS →
B_{10} Life = 1,121 Hrs.
Weibull Slope = 2.83
(N_1 = 5)

Figure 2

Weibull Plot of Test Sample # 2

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