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LOGISTIC PLOTTING OF MEASUREMENTS IN QUALITY CONTROL, USING TABLES OF ODDS AGAINST EXCEEDANCE FOR ORDER STATISTICS RESULTING FROM ARRANGING THE OBSERVED MEASURED VALUES INTO NUMERICAL ORDER

INTRODUCTION

The graphical plotting of cumulative distribution functions has been found to be very informative and easily interpreted for the purposes of analyzing the important parameters of the population to which a sample of measurements is estimated to belong. Examples of such plotting are normal distribution plots on normal probability paper and Weibull plots on Weibull probability paper.

In this bulletin we are presenting a method of graphically plotting measurements taken in a quality control program and fitting the data to a Logistic type of distribution function. The type of graph paper used is semi-log paper, with the horizontal axis representing the values of the measurements in a random sample taken from a factory process on an assembly line or a test laboratory, and the vertical axis representing the odds against exceeding each measured value, after all the measured values have been made into order statistics by putting them into numerical order (smallest to largest). If the data set lines up straight, with a high goodness of fit, this indicates that a Logistic Distribution Function is appropriate as a representation of the data set's population. The slope of the line (with scaling taken into consideration) is then the Logistic slope for the distribution function, and the point with Even Odds (1 to 1) has the Distribution Mean as its abscissa. From the Logistic slope we can determine the Logistic Sigma by simply dividing $\pi/\sqrt{3}$ by the Logistic slope. All this will be clarified by an actual example.

A TYPICAL EXAMPLE

A steel shaft is being produced on a factory assembly line, and is to be in compliance with the following specifications:

Lower Specification Limit (LSL) = 1.950 in. (diameter)
Nominal Value (NV) = 2.000 in. (diameter)
Upper Specification Limit (USL) = 2.050 in. (diameter)

An inspector removes 8 shafts off the assembly line during a particular hours of the day. The eight shafts have the following diameters:

2.007, 1.997, 1.996, 2.004, 2.002, 2.010, 1.995, and 1.999

Arranging these 8 measurements into numerical order, we obtain the following table:

ORDER NO.	SHAFT DIAMETER	ODDS AGAINST EXCEEDANCE (from the TABLE for N=8)
1	1.995	0.0909
2	1.996	0.2537
3	1.997	0.4737
4	1.999	0.7872
5	2.002	1.2700
6	2.004	2.1110
7	2.007	3.9410
8	2.010	11.0000

Next, we plot this data set on semi-log paper, taking Diameters as abscissas and the Exceedance Odds as ordinates. The resulting graph is shown in Figure 1, with the best fitting line drawn through the set of points. This best fitting line is the estimated Logistic Distribution Function of the population of shaft diameters, with the two parameters as follows:

Parameter # 1 : The Logistic Mean (at unit odds)

Parameter # 2 : The Logistic Slope
Found by taking any two points (X_1, Y_1) and (X_2, Y_2) and calculating:

$$\text{SLOPE} = \frac{Y_2 - Y_1}{X_2 - X_1}$$

X = Measured Diameter
Y = ln(Exceedance Odds)

In this example:

Logistic Mean = 2.001
Logistic Slope = 271.7

From the graphical plot on semi-log paper we can now determine (from odds) what fraction of the population will fall between the specification limits, as well as the amount of shift from the Nominal Value to the Mean Value.

The best way to make such predictions is to use DRI's "LOGIPAR" Computer Program, discussed in our August 1992 bulletin, i.e., Volume 22, Bulletin 4.

(See computer output on Page 6.)

TABLE OF ODDS AGAINST EXCEEDANCE FOR ORDER STATISTICS

	SAMPLE SIZE									
	N=1	N=2	N=3	N=4	N=5	N=6	N=7	N=8	N=9	N=10
1	1.0000	.4118	.2593	.1892	.1489	.1228	.1045	.0909	.0805	.0722
2		2.429	1.000	.6296	.4595	.3617	.2982	.2537	.2208	.1954
3			3.857	1.588	1.000	.7297	.5745	.4737	.4030	.3506
4				5.286	2.176	1.370	1.000	.7872	.6491	.5522
5					6.714	2.765	1.741	1.270	1.000	.8246
6						8.143	3.353	2.111	1.541	1.213
7							9.571	3.941	2.481	1.811
8								11.00	4.529	2.852
9									12.43	5.118
10										13.86

	SAMPLE SIZE									
	N=11	N=12	N=13	N=14	N=15	N=16	N=17	N=18	N=19	N=20
1	.0654	.0598	.0551	.0511	.0476	.0446	.0419	.0395	.0374	.0355
2	.1753	.1589	.1453	.1339	.1241	.1156	.1083	.1018	.0960	.0909
3	.3103	.2784	.2523	.2308	.2126	.1971	.1837	.1720	.1617	.1525
4	.4805	.4253	.3814	.3458	.3162	.2913	.2701	.2517	.2357	.2216
5	.7015	.6104	.5402	.4845	.4393	.4017	.3701	.3431	.3197	.2994
6	1.000	.8507	.7403	.6552	.5876	.5327	.4872	.4488	.4161	.3878
7	1.426	1.175	1.000	.8701	.7701	.6907	.6262	.5726	.5276	.4891
8	2.081	1.638	1.351	1.149	1.000	.8851	.7938	.7196	.6581	.6063
9	3.222	2.351	1.851	1.526	1.299	1.130	1.000	.8969	.8131	.7436
10	5.706	3.593	2.622	2.064	1.702	1.448	1.260	1.115	1.000	.9065
11	15.29	6.294	3.963	2.892	2.277	1.877	1.597	1.390	1.230	1.103
12		16.71	6.882	4.333	3.162	2.489	2.053	1.746	1.519	1.345
13			18.14	7.471	4.704	3.432	2.702	2.228	1.896	1.649
14				19.57	8.059	5.074	3.703	2.915	2.404	2.045
15					21.00	8.647	5.444	3.973	3.128	2.579
16						22.43	9.235	5.815	4.243	3.340
17							23.86	9.824	6.185	4.514
18								25.29	10.41	6.556
19									26.71	11.00
20										28.14

STEEL SHAFT EXAMPLE

SEMI-LOG PLOT OF EXAMPLE

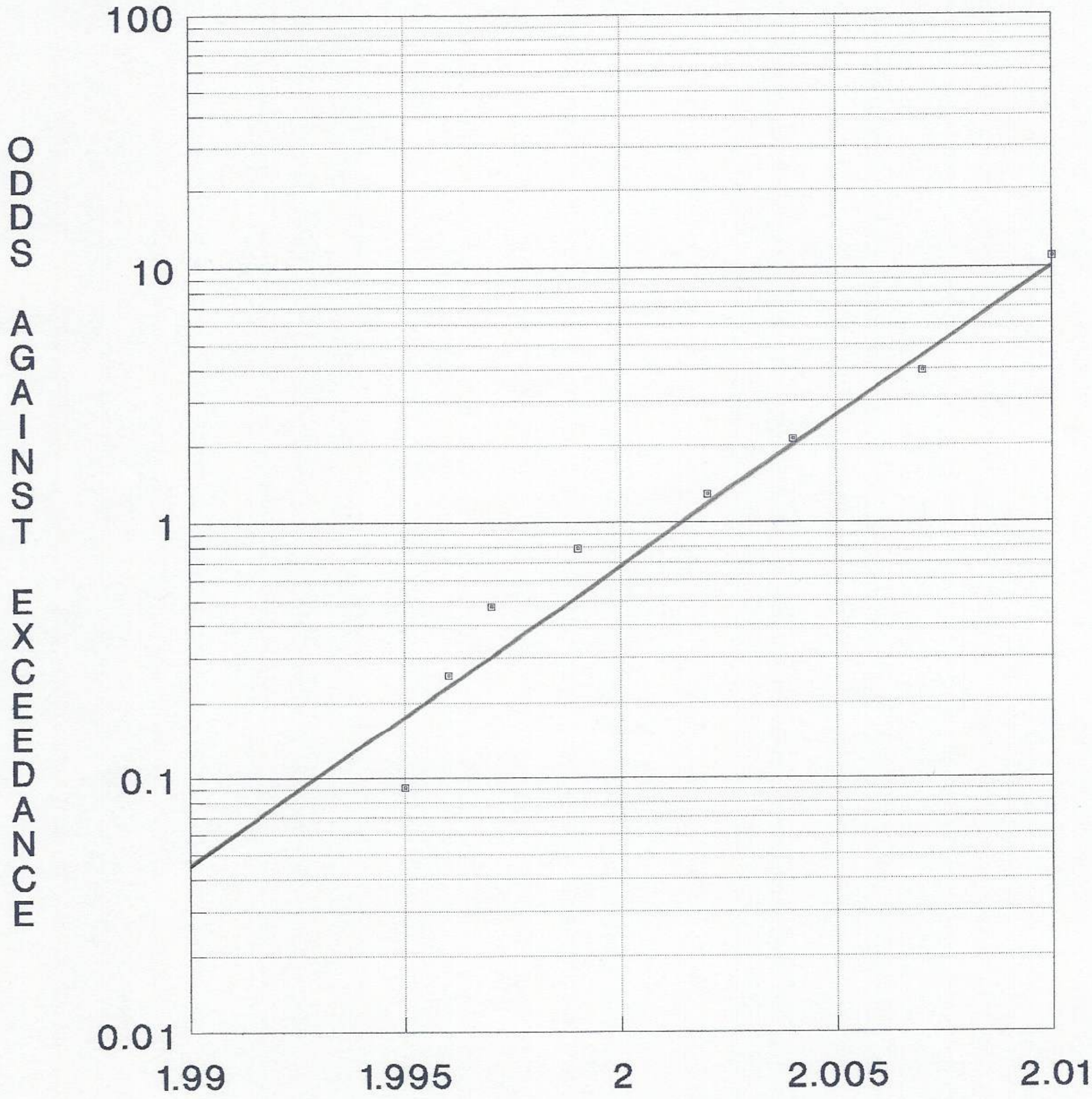

Figure 1

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PROBLEM TITLE: LOGISTIC PARAMETERS BY LEAST SQUARE

NO. OF MEASUREMENTS =  8
J= 1 AND X= 1.995 ODDS AGAINST EXCEEDENCE =  9.090909E-02 TO 1 (AT X =  1.995 )
J= 2 AND X= 1.996 ODDS AGAINST EXCEEDENCE =  .2537314 TO 1 (AT X =  1.996 )
J= 3 AND X= 1.997 ODDS AGAINST EXCEEDENCE =  .4736843 TO 1 (AT X =  1.997 )
J= 4 AND X= 1.999 ODDS AGAINST EXCEEDENCE =  .7872341 TO 1 (AT X =  1.999 )
J= 5 AND X= 2.002 ODDS AGAINST EXCEEDENCE =  1.27027 TO 1 (AT X =  2.002 )
J= 6 AND X= 2.004 ODDS AGAINST EXCEEDENCE =  2.111112 TO 1 (AT X =  2.004 )
J= 7 AND X= 2.007 ODDS AGAINST EXCEEDENCE =  3.941177 TO 1 (AT X =  2.007 )
J= 8 AND X= 2.01 ODDS AGAINST EXCEEDENCE =  11 TO 1 (AT X =  2.01 )

                                   LOGISTIC SLOPE =  271.6932

                                   LOGISTIC MEAN =  2.00125

                                   GOODNESS OF FIT =  .9690416

                                   LOGISTIC SIGMA =  6.675913E-03

     LOWER SPEC =  1.95

     NOMINAL VALUE =  2

     UPPER SPEC =  2.05

         % BELOW LOWER SPEC =  8.969459E-05 %

         % BELOW NOMINAL VALUE =  41.59007 %

         % ABOVE UPPER SPEC =  1.76913E-04 %

         DISTRIBUTION'S MEAN SHIFT FROM NOMINAL =  1.250029E-03

         PERCENT WITHIN THE TWO SPECS =  99.99972 %

         PERCENT OUTSIDE THE TWO SPECS =  2.666076E-04 %
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CONCLUSION

It can be seen that the Semi-log plotting of measurements in quality control is a very convenient way of picturing the underlying population to which any sample of measurements belongs. The Goodness of Fit tells how valid it is to accept such an estimated population line.