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# RELIABILITY AND CONFIDENCE REQUIREMENTS AS DICTATED BY ECONOMIC FACTORS

## INTRODUCTION

In a capitalistic economy, consisting of private enterprises in competition with one another, the business manager must concern himself with profits and losses. For this simple reason, product reliability and the confidence of attaining it must take profits and losses into account. Insufficient reliability causes excessive losses when warranty promises are violated too frequently, thus putting a business in jeopardy. On the other hand, aiming for absolutely perfect reliability can be so expensive that the price we must charge for our product to make any money becomes so high that competitors with lower prices and acceptable non-perfect reliability, will take away our customers. What all this amounts to is the fact that there are optimum reliability and confidence levels, which are dictated by possible gains and losses. This, in turn, means that the test sample sizes for reliability are also determined by the economic factors of profits and losses.

The purpose of this bulletin is to clarify and systematize the whole concept of the role of profits and losses in product reliability developmental programs needed in qualifying a product before it is finally released for production and sold to the consumer to generate an acceptable profit margin. Thus, it actually turns out that the degree of profitability, as defined by the ratio of (Gains/Losses), should be the fundamental criterion for the reliability required, together with the proper confidence level, as the determining factor for the sample size required in a life test.

## OUTLINE OF THE CRITICAL CONCEPTS

### I : GAINS AND LOSSES

(A) By GAINS we mean the net profit generated by good items, which are reliable, i.e., which survive the product life goal.

(B) By LOSSES we mean the cost of having a bad item sold to a customer, i.e., the loss due to all required replacements and repairs on an item which does not survive the promised life under warranty.

### II : RELIABILITY AND FAILURE PROBABILITY

(A) By RELIABILITY we mean the fraction of the population of items sold which will survive a desired service time without failure.

(B) By FAILURE PROBABILITY we mean the fraction of the population of items sold which will fail (at least once) in a desired service time.

### III: MEASURING THE MAGNITUDE OF A FAILURE PROBLEM (FOR A SOLD SYSTEM)

(A) When dealing with a REPAIRABLE SYSTEM we can measure the seriousness of a system failures in two ways:

(1) We can simply count the failures per system in a specific service time. This is known as the ENTROPY for that type of system in the specified service time.

(2) We can count up the DOLLAR LOST in correcting system failures in a specific service time. This is known as the DOLLAR ENTROPY for the system in the specified service time.

## IV: MATHEMATICAL FUNCTIONS USED IN THE EVALUATION

(A) x = Service Time

(B) R(x) = Reliability to Service Time x

(C) F(x) = Fraction Failed (at least once) in Service Time x
= 1 - R(x)

(D) &(x) = Failures per System in time x
= Entropy to time x
= -ln R(x)
= Failure Rate to time x

## V: THE PROFITABILITY RATIO

(A) The PROFITABILITY RATIO for a product with Reliability R(x) and a Failure Probability F(x) is defined in terms of :

(1): The Gain per Good Item

(2): The Loss per Bad Item

A **Good Item** is a reliable one to the desired service time x .

A **Bad Item** is one which fails (at least one) in the desired service time x.

We let G = GAIN PER GOOD ITEM

and L = LOSS PER BAD ITEM

So, the PROFITABILITY RATIO K(x) to service time x is

$$K(x) = \frac{G\,R(x)}{L\,F(x)} = \frac{G\,R(x)}{L[1 - R(x)]}$$

## A TYPICAL SITUATION

As an example of applying the basic economic principles for product reliability, consider the following situation :

Desired Life = 1000 Hours

Item Cost to Manufacturer = \$500

Item's Selling Price = \$600

Gain per Good Item = \$600 - \$500 = \$100 = G

Loss per Bad Item = Cost of Replacement Item - Original Profit
= \$500 - \$100 = \$400 = L
(Replacement given free of charge to customer)

Desired Profitability Ratio = 4
(i.e., the manufacturer wants the gains to be 4 times the losses)

So,

$$K(1000) = \frac{100\ R(1000)}{400\ [1 - R(1000)]} = 4$$

or, $R(1000) = 16[1 - R(1000)]$

from this, $R(1000) = 16/17 = .94118$

So, to gain 4 times as much from reliable items as is lost from unreliable ones, the product reliability must be .94118.

## THE QUESTION OF CONFIDENCE

In the example just considered we found that the reliability to 1000 hours must be .94118 in order to realize gains 4 times as large as losses. The next question is HOW MUCH BETTER THAN .94118 MUST THE RELIABILITY BE ON A TEST WEIBULL PLOT FROM A SAMPLE OF SIZE **N** IN ORDER TO BE CERTAIN (100% CONFIDENT) THAT THE PROFITABILITY RATIO IS AT LEAST 4:1 ? Since a Weibull plot of test data based on median ranks yields values of reliability at the 50% confidence level, it is necessary that the Weibull plot of the test data shows more reliability (at 50% confidence) than .94118 (at 100% confidence). Then profitability ratio above 4:1 will show confidence indices somewhere below 100%. This whole problem is handled by the use of ENTROPY, as shown in the next section.

## PROFITABILITY CONFIDENCE VIA ENTROPY

The natural logarithm of entropy is, for all practical purpose, log-normally distributed. If we want the UPPER 3 SIGMA LIMIT of the natural logarithm of the entropy to be at

$$\ln \ln (1/.94118) = -2.80312 ,$$

with a STANDARD DEVIATION of $1/\sqrt{.5N}$ , then the observed ln(ENTROPY) in the test data must be $-2.80312 - 3/\sqrt{.5N}$ . This will then mean that we are practically certain of a reliability of at least .94118 for our product.

Suppose that a sample of N items shows 1% failed at 1000 hours on a Weibull plot. Then the required value of N must be large enough to make <u>3 SIGMAS</u> of ln(ENTROPY) equal to

$$-2.80312 - \ln \ln(1/.99) = 1.79703 ,$$

or, $\text{SIGMA of } \ln(\text{Entropy}) = 1.79703/3 = .59901 = 1/\sqrt{.5N}$

So, $\sqrt{.5N} = 1/.59901 = 1.66942$ , or $.5N = 2.78696$ .

Thus , the required sample size is $N = 2(2.78696) = 5.57392$ , or, to the NEXT INTEGER , the required sample size is <u>N = 6</u>.

The PROFITABILITY RATIO K(x) is $K(x) = \frac{G\,R(x)}{L[1 - R(x)]}$ ,

and whatever confidence R(x) possesses is transferred to K(x) .

Thus, in the numerical example which we are considering, we have

$$\text{SIGMA}_{\ln \mathcal{E}} = 1/\sqrt{.5N} = .57735 \quad (\text{For } N = 6)$$

$$(\mathcal{E} = \text{Entropy} = -\ln R\text{ , with } R = \text{Reliability})$$

The observed entropy at 1% failed in 1000 hours is -ln(.99) = .01005

The natural logarithm of this observed entropy is ln(.01005)=-4.60015

1 SIGMA (at 84% confidence) puts ln(Entropy) at -4.60015 + .57735 =

= -4.0228 or ENTROPY = exp(-4.0228) = .01790.

Thus, the RELIABILITY corresponding to ± 1 SIGMA is

$$\exp(-.01790) = .98226 \quad .$$

Then, the corresponding profitability ratio (at 84% confidence) is

$$K(x) = \frac{100(.98226)}{400(.01774)} = 13.84$$

At the observed RELIABILITY of .99 the corresponding profitability factor (at 50% confidence) is

$$K(x) = \frac{100(.99)}{400(.01)} = 24.75$$

## COMPUTER PROGRAM NOTE

All the calculations needed to construct a confidence interpolation diagram for profitability ratios can be performed automatically by employing a modification of the PROFCON program given in the MAY 1991 BULLETIN (Vol. 21, #2). Instead of using PERCENT PROFIT (OF SELLING PRICE), we use PERCENT PROFIT (OVER MFG. COST). This modified program is called "MODPRCF" .

## THE CONFIDENCE INTERPOLATION DIAGRAM

A confidence interpolation diagram for the profitability factor can now be constructed, as shown by the graph in Figure 1 . This confidence interpolation diagram then becomes our final reference criterion for what kind of profitability ratios can be predicted with different degrees of confidence, including the ORIGINALLY PROPOSED RATIO (4:1 in the example) of which we are practically certain (since it is at the 3 SIGMA LEVEL) when we (in the example) test 6 items to produce a Weibull plot which crosses 1% failed at the target of x = 1000 hours.

### PARAMETER VALUES FROM THE EXAMPLE :

Observed Reliability from Test = .99
Test Sample Size = 6
Percent Profit (Over Mfg. Cost) = 20%