Statistical Bulletin
Reliability & Variation Research

DETROIT RESEARCH INSTITUTE
P.O. BOX 36504 • GROSSE POINTE, MICHIGAN 48236 • (313) 886-7976

LEONARD G. JOHNSON
EDITOR

WANG H. YEE
DIRECTOR

Volume 21
Bulletin 3

July , 1991

COMPUTER PROGRAMS FOR ANSWERING BASIC QUESTIONS IN WEIBULL ANALYSIS
(As told in a discussion between Mr. Manager and Mr. Analyst)

PREFACE

The modern way to arrive at quantitative answers in engineering statistics is to use a computer. In particular, this is certainly true of what is known as Weibull Analysis. In this bulletin we shall consider the five most common questions relating to Weibull Analysis and the corresponding computer programs for answering them. These five questions are:

(1) How is the best fitting line for a data set determined by using median ranks and least squares on Weibull paper ? (The computer program "LEASQ")

(2) How do we construct a 90% confidence band about a Weibull plot ? (The computer program "CRPRG")

(3) How do we calculate the confidence of meeting a specific life goal, such as a required B-10 life ? (The computer program "GOALCNF")

(4) How do we compare two separate Weibull plots at a particular level, such as the B-10 level ? (The computer program BQCNF")

(5) What sample size will make the confidence band of a data plot narrow enough ? (The computer program "SSCBAND")

These five questions are answered, with examples, in an interesting discussion between Mr. Manager and Mr. Analyst. All five computer programs are available on a disk sold by DRI for use with IBM Personal Computers and Compatibles.

MR. MANAGER LEARNS WEIBULL ANALYSIS FROM MR. ANALYST

(A DISCUSSION OF A MOST AMAZING STATISTICAL TOOL)

INTRODUCTION

Mr. Manager was in charge of the product development testing lab. of XYZ Corporation. Because of his position he sincerely wanted to keep up with the latest analytical techniques for effectively interpreting test data on product reliability, where decisions are of utmost importance and do affect the survival of a business. He had heard much talk about the Weibull Analysis of life test data and was wondering what it was all about.

He had just hired Mr. Analyst, a capable reliability engineer who was fully familiar with Weibull Analysis. During the first week that Mr. Analyst was working in the test lab. Mr. Manager called Mr. Analyst into his front office with the intention of familiarizing himself with the fundamentals of Weibull Analysis. The discussion which ensued between Mr. Manager and Mr. Analyst is reported in detail in the following pages.

QUESTION #1: MAKING WEIBULL PLOTS

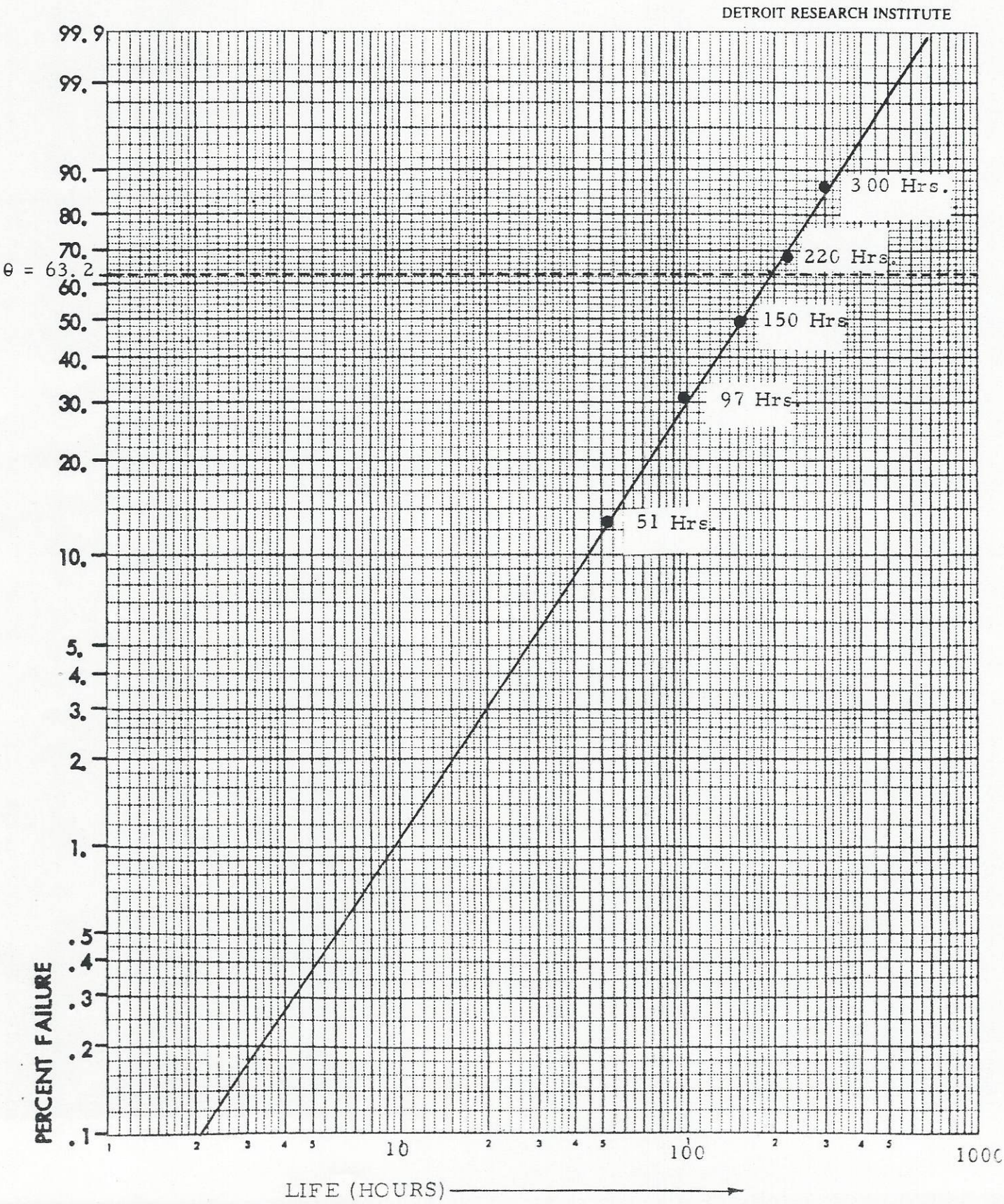
Mr. Manager: I wish to welcome you, Mr. Analyst, into our busy testing laboratory. I am sure that you can truly help us build a successful and profitable business by means of effectiveness statistical procedures applied to test data obtained in our lab, as well as from prototype tests and actual customer experiences in the field.

Mr. Analyst: I'm very anxious to get started on the systematic and effective statistical control and interpretation of all testing programs in this lab.

Mr. Manager: My first question is "What is a Weibull plot of life test data, and how is it constructed ?"

Mr. Analyst: A Weibull plot of life test data is a graph on Weibull paper which turns out to be a straight line if the data come from a Weibull population. The horizontal axis (abscissa) represents the length of life and the vertical axis (ordinate) represents the fraction of the population failed.

WEIBULL PLOT OF LIFE TEST DATA



Mr. Manager: I see on the plot that there is a sample of 5 failures (51 Hrs., 97 Hrs., 150 Hrs., 220 Hrs., 300 Hrs.). How are the ordinates for these failures determined ?

Mr. Analyst: Failure #1 in 5 (first order statistic in a sample of 5) is placed at 12.96% failed (Median Rank of 1st of 5)
Failure #2 in 5 (second order statistic in a sample of 5) is placed at 31.48% failed (Median Rank of 2nd of 5)
Failure #3 in 5 (third order statistic in a sample of 5) is placed at 50% failed (Median Rnk of 3rd of 5)
Failure #4 in 5 (fourth order statistic in a sample of 5) is placed at 68.52% failed (Median Rank of 4th of 5)
Failure #5 in 5 (fifth order statistic in a sample of 5) is placed at 87.04% failed (Median Rank of 5th of 5).

The Median Rank of an order statistic (order life) is the median location it has in the underlying population i.e., half of all possible samples of the same size (5 in this case) would show that order statistic below the Median Rank, while half of all possible samples of the same size would show order statistic above the Median Rank. Therefore, the Median Rank is considered the best estimate of the location in the population for the order statistic. The formula for the Median Rank of Order Statistic #J in a sample of size N is given by the formula

$$\frac{J - .3}{N + .4}$$ (Benard's Formula)

QUESTION #2:
THE BEST FITTING LINE THROUGH THE PLOTTED POINTS

Mr. Manager: Once I have plotted individual Time to Failure by using Median Ranks, how do I draw the best fitting line through the plotted points ?

Mr. Analyst: You use what statisticians called Least Squares Regression Analysis. There is a computer program in BASIC language for doing this. It is called **"LEASQ"**. Once you have this program loaded into your computer's memory you tell it to "RUN". It then asks the following questions :

			ENTER	PRINTOUT
Min. Life		= ?	0	
Total Sample Size		= ?	5	
Total No. Failed		= ?	5	
No. Susp. in Interval	1	= ?	0	
No. Failed in Interval	1	= ?	1	
End Pt. of Interval	1	= ?	51	.1296
No. Susp. in Interval	2	= ?	0	
No. Failed in Interval	2	= ?	1	
End Pt. of Interval	2	= ?	97	.3148
No. Susp. in Interval	3	= ?	0	
No. Failed in Interval	3	= ?	1	
End Pt. of Interval	3	= ?	150	.5000
No. Susp. in Interval	4	= ?	0	
No. Failed in Interval	4	= ?	1	
End Pt. of Interval	4	= ?	220	.6852
No. Susp. in Interval	5	= ?	0	
No. Failed in Interval	5	= ?	1	
End Pt. of Interval	5	= ?	300	.8704

Weibull Slope	= 1.4875
Theta	= 191.126
Goodness of Fit	= .99913
Std. Error of Slope b	= .4704
B10	= 42.100
B50	= 149.386
B90	= 334.834

NOTE: Weibull Slope (b) is the slope of the line
Theta is where 63.2% has failed
Goodness of Fit is the Correlation Coefficient
B10 is where 10% has failed
B50 is where 50% has failed
B90 is where 90% has failed

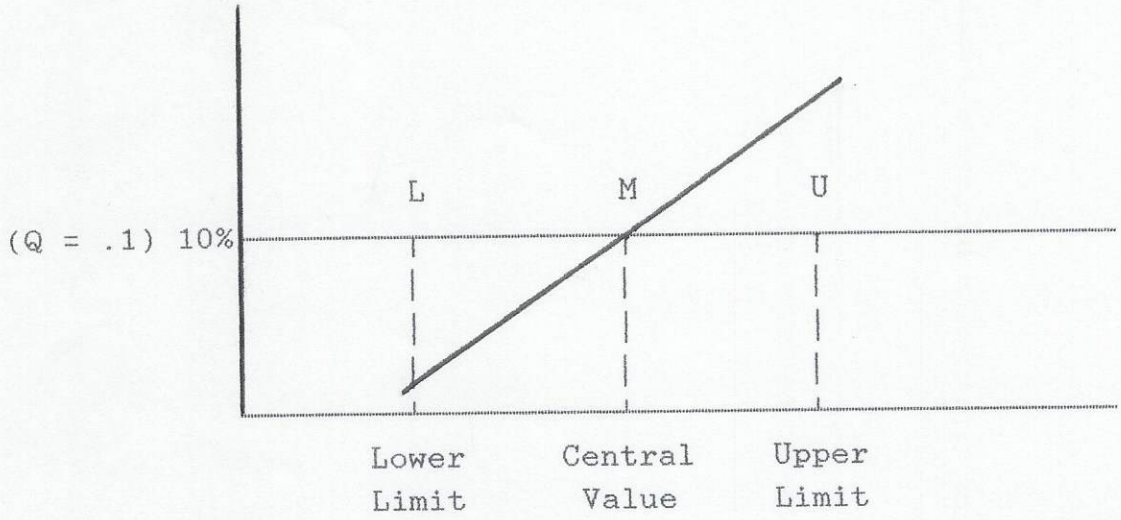
QUESTION #3: CONSTRUCTING A 90% BAND ABOUT A WEIBULL PLOT

Mr. Manager: O. K., now that we have drawn the best fitting line for the data plot, how do we put a 90% confidence band around the line ?

Mr. Analyst: We make use of the Computer Program called "CRPRG" which is the Common Ratio Program at any Quantile Level Q (e.g., Q = .1 at B10 life, where 10% has failed). We determine the Common Ratio

$$\frac{\text{Central Value}}{\text{Lower Limit}} = \frac{\text{Upper Limit}}{\text{Central Value}}$$

Once the Computer Program determines this ratio for the sample size we have. We can determine L and U for the specific Quantile Level, such that we are 90% confident that the Bq Life is in the Interval from L to U.



Once this program is in your computer's memory, you tell it to "RUN". Then it asks Input questions :

		ENTER
Min. Life	= ?	0
Weibull Slope	= ?	1.4875
Quantile Level	= ?	.1
Sample Size	= ?	5
Bq Life	= ?	42.1

PRINTOUT

Common Ratio = 1.928

L = 21.835 M = 42.1 U = 81.171

(Thus, at the 10% Level the 90% Confidence Band is from 21.835 Hrs. to 81.171 Hrs.)

QUESTION #4: CONFIDENCE OF MEETING A SPECIFIC LIFE GOAL

Mr. Manager: Alright, now suppose I want to guarantee a B10 Life of at least 15 hours. How much Confidence does the data set provide for meeting this goal of 15 hours for B10 Life ?

Mr. Analyst: To answer your question we employ the Computer Program "GOALCNF" (Goal Confidence). Load the program into your computer memory and tell it to "RUN". The program asks the following questions :

		ENTER
Quantile Level	= ?	.1
Bq Goal at Quantile Level	= ?	15
Sample Weibull Slope	= ?	1.4875
Sample Bq Life	= ?	42.1
Sample Size at Bq Life	= ?	5

PRINTOUT

Confidence of Meeting

Bq Goal = .990325

Thus, this data assures us with 99.0% that the B10 Life Goal of 15 Hours will be met.

QUESTION #5: COMPARING TWO SEPARATE WEIBULL PLOTS

Mr. Manager: Now suppose I think I have a superior new design with a sample of 4 life values 95 Hrs., 165 Hrs., 310 Hrs., 470 Hrs. How confident can I be that the new design's B10 Life is better than the first design's B10 Life we calculated for Question #2 ?

Mr. Analyst: You first use "LEASQ" and find that your new design has a Weibull slope of 1.4248 and a B10 Life of 63.0 Hrs. Now run Program "BQCNF". It asks the following questions:

		ENTER
Quantile Level	= ?	.1
1st Weibull Slope	= ?	1.4875
1st Bq Life	= ?	42.1
1st Sample size	= ?	5
2nd Weibull Slope	= ?	1.4248
2nd Bq Life	= ?	63.0
2nd Sample Size	= ?	4

PRINTOUT

Confidence that
2nd Bq > 1st Bq
= .76456

Thus, Mr. Manager, you can be 76.456% Confident of having an improved B10 Life with your new design. If you want more confidence you must increase the sample size.

QUESTION #6:

WHAT SAMPLE SIZE WILL MAKE THE CONFIDENCE BAND NARROW ENOUGH ?

Mr. Manager: In Question #3 the Ratio (Central Value/Lower Limit) was 1.928 at B10. If I want this ratio reduced down to 1.414 (So that the Upper Limit is **twice** the Lower Limit), what sample size would I need for the same slope as in the example of Question #1 ?

Mr. Analyst: You must use the Computer Program "SSCBAND" (Sample Size for Confidence Band). Load it into your computer's memory and tell it to "RUN". Then it asks the following questions :

	ENTER
Quantile Level = ?	.1
Weibull Slope = ?	1.4875
Total Band Ratio = ?	2

PRINTOUT

Theoretical N = 17.94169

Required Sample = 18

So, you see, Mr. Manager, for a 2:1 Band Ratio the test sample size must be 19 instead of just 5.

CONCLUDING REMARKS

Mr. Manager: Thank you, Mr. Analyst. You have given me a good idea of what Weibull Analysis is all about. You have answered the main questions I had about the procedures and computer programs to be used. Thank you very much. I'm sure we can work out a very effective program for our lab.

Mr. Analyst: You're welcome. I'm looking forward to a great career in this testing laboratory.