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DESIGNING LIFE TESTS WITH SAMPLE SIZES SUFFICIENT TO ASSURE A DESIRED DEGREE OF PROFITABILITY

INTRODUCTION

The durability of consumer products such as machines or vehicles and their components and sub-assemblies are expected to last for a certain warranty period promise, which, if violated, requires free replacement or repair service to the customer. Because of the risk of excessive failures earlier than the promised warranty period there is the possibility of warranty replacement and repair expenses becoming so large that they exceed profits from sales. Due to this fact of life in the management of product assurance it is absolutely necessary that product life tests are properly designed with sufficiently large sample sizes to assure a desired degree of profitability as expressed by the ratio

$$\frac{\text{PROFIT FROM SALES}}{\text{LOSSES DUE TO WARRANTY FAILURES}}$$

Life tests must be designed to make this profitability ratio sufficiently greater than unity to whatever extent is feasible considering limitations of lead time and test facilities

In this bulletin we shall present the general theory of designing proper life tests and sample sizes so as to assure a desired value for profitability ratio by considering the following factors:

Factor # 1: The Profitability Ratio Desired

Factor # 2: The Percent of Profit (of Selling Price) Per Item Sold

Factor # 3: The Observed Reliability From The Test (i.e., Fraction Surviving The Warranty Target) (in the Test as Shown by a Weibull Plot.)

Factor # 4: The Test Sample Size

Factor # 5: The Confidence of Realizing the Desired Profitability Ratio

Based on these factors we shall present two computer programs.

These are:

(1) The Program "PROFCON" for the confidence of realizing the desired profitability ratio with a life test of specific sample size.

(2) The Program "SAMPSIZ" for the sample size required when we have a specific safety factor for the needed reliability

THE PROFITABILITY CONFIDENCE PROGRAM ("PROFCON")

The calculation of the confidence of realizing a profitability factor P_c (with confidence c) is done by evaluating the formula

$$\text{ODDS} = \left\{ \frac{\ln\left[1 + \dfrac{P}{P_c(100 - P)}\right]}{\ln\left(1/R_{obs.}\right)} \right\}^{\sqrt{\frac{N(1 - .5\,R_{obs.})}{.304}}}$$

Then , $$\text{CONFIDENCE} = \frac{\text{ODDS}}{1 + \text{ODDS}}$$

In the above formula we define the parameters as follows:

P = Percent Profit (of Selling Price)

P_c = Desired Profit Ratio = $\dfrac{\text{Profits from Sales}}{\text{Warranty Dollar Loss}}$

$R_{obs.}$ = Observed Reliability at Target (From Weibull Plot)

N = Life Test Sample Size

NOTE: $.304 = 3/\pi^2$

THE SAMPLE SIZE PROGRAM ("SAMPSIZ")--(FOR RELIABILITY TO A TARGET)

L = Loss Per Bad Item

G = Gain Per Good Item

M = Money Ratio = L/G

Pc =Profitability Ratio Desired with Confidence c.

Then, the reliability required with confidence c is

$$R_c = \frac{P_c M}{1 + P_c M}$$

If a reliability $R_{obs.} > R_c$ is observed in a Weibull plot of N failures, then in order to realize the profitability ratio Pc , we must have sample size

$$N = \left[\frac{.304}{1 - .5R_{obs.}}\right]\left[\frac{\ln[c/(1-c)]}{\ln\ln(1 + 1/P_cM) - \ln\ln(1/R_{obs.})}\right]^2$$

NOTES:

(1) $.304 = 3/\pi^2$

(2) $R_{obs.}$ must exceed $R_c = \frac{P_c M}{1 + P_c M}$

THE TWO CRITICAL QUESTIONS IN LIFE TESTING

Consumer products which must be tested for durability, that is, for reliability to some time target or length of usage, present management with two distinct problems, which we shall call the two critical questions in life testing. Both of these questions deal with survivability. First and foremost, any responsible manager dealing with life testing for product assurance must ask himself the most important question of all:

QUESTION # 1: **(Profit Assurance)** How can I design a life test with a sufficiently large sample to assure a desired level of profitability when I make a warranty promise to my customers?
This is the question of **BUSINESS SURVIVABILITY.**

QUESTION # 2: **(Reliability Assurance)** What is the reliability level of my product to a warranty goal, and how do I measure it from life tests? This is the question of **PRODUCT SURVIVABILITY.**

These two questions, instead of being independent, bear a definite relationship to one another. Obviously, a more reliable product produces fewer failure expenses to the promised warranty life. However, there is such a thing as a level of reliability so exceedingly high that it is not feasible when we consider the developmental lead time needed, as well as the present state of the art for the type of product we must produce.

The two computer programs we here present, i.e., "PROFCON" and "SAMPSIZ", will evaluate any life test under consideration by indicating the confidence for any desired profitability ratio, as well as the sample size needed when the test results plotted on Weibull paper show a specific level of reliability to a desired endurance target. What makes this approach especially useful is that it doesn't matter what type of Weibull plot the data yield, since we are looking at the Weibull ordinate, which is valid for curves as well as straight lines as far as any warranty loss from a bad item which fails prior to service target (or the equivalent lab test target) is concerned. We assign a loss equal to the total product cost for its manufacture and delivery, including the labor cost involved. We also have the percent profit (of selling price) as an input parameter to obtain the gain per good item.

LISTINGS OF COMPUTER PROGRAMS OF "PROFCON" AND "SAMPSIZ"

```
10 PRINT TAB(20)"PROFITABILITY CONFIDENCE PROGRAM (PROFCON)"
20 PRINT:PRINT
30 LPRINT TAB(20)"PROFITABILITY CONFIDENCE PROGRAM (PROFCON)"
40 LPRINT:LPRINT
50 INPUT "OBSERVED RELIABILTY FROM TEST";R
60 LPRINT "OBSERVED RELIABILTY FROM TEST =";R
70 INPUT "TEST SAMPLE SIZE";N
80 LPRINT "TEST SAMPLE SIZE =";N
90 INPUT "DESIRED PROFITABILITY RATIO";P
100 LPRINT "DESIRED PROFITABILITY RATIO =";P
110 INPUT "PERCENT PROFIT (OF SELLING PRICE)";Q
120 PRINT:PRINT:PRINT
130 LPRINT "PERCENT PROFIT (OF SELLING PRICE) = ";Q
140 LPRINT:LPRINT:LPRINT
150 A = LOG(1+Q/(P*(100 - Q)))
160 B = LOG(1/R)
170 E = SQR((N/.304)*(1-.5*R))
180 D = (A/B)^E
190 C = D/(1+D)
200 PRINT TAB(10)"CONFIDENCE FOR DESIRED PROFITABILITY RATIO = ";C
210 LPRINT TAB(10)"CONFIDENCE FOR DESIRED PROFITABILITY RATIO = ";C
220 END
```

```
10 PRINT TAB(30)"SAMPLE SIZE PROGRAM (SAMPSIZ)"
20 PRINT:PRINT
30 LPRINT TAB(30)"SAMPLE SIZE PROGRAM (SAMPSIZ)"
40 LPRINT:LPRINT
50 INPUT "DESIRED PROFITABILITY RATIO";P
60 LPRINT "DESIRED PROFITABILITY RATIO=";P
70 INPUT "PERCENT PROFIT (OF SELLING PRICE)";Q
80 LPRINT "PERCENT PROFIT (OF SELLING PRICE)=";Q
90 INPUT "FAILURE RATE (ENTROPY) SAFETY FACTOR";E
100 LPRINT "FAILURE RATE (ENTROPY) SAFETY FACTOR=";E
110 INPUT "CONFIDENCE DESIRED FOR PROFITABILITY RATIO";C
120 LPRINT "CONFIDENCE DESIRED FOR PROFITABILITY RATIO=";C
130 PRINT:PRINT:PRINT
140 LPRINT:LPRINT:LPRINT
150 A = (P*(100-Q))/(Q+P*(100-Q))
160 B = LOG(C/(1-C))/LOG(E)
170 D = 1-.5*(A^(1/E))
180 N = (.304/D)*B*B
190 N = 1 + INT(N)
200 PRINT TAB(10)"REQUIRED SAMPLE SIZE = ";N
210 LPRINT TAB(10)"REQUIRED SAMPLE SIZE = ";N
220 END
```

EXAMPLES OF USING THE "PROFCON" PROGRAM

PROFITABILITY CONFIDENCE PROGRAM (PROFCON)

OBSERVED RELIABILTY FROM TEST = .98982
TEST SAMPLE SIZE = 18
DESIRED PROFITABILITY RATIO = 2
PERCENT PROFIT (OF SELLING PRICE) = 3

CONFIDENCE FOR DESIRED PROFITABILITY RATIO = .9017231

PROFITABILITY CONFIDENCE PROGRAM (PROFCON)

OBSERVED RELIABILTY FROM TEST = .99231
TEST SAMPLE SIZE = 10
DESIRED PROFITABILITY RATIO = 1.5
PERCENT PROFIT (OF SELLING PRICE) = 2

CONFIDENCE FOR DESIRED PROFITABILITY RATIO = .9071691

PROFITABILITY CONFIDENCE PROGRAM (PROFCON)

OBSERVED RELIABILTY FROM TEST = .999369
TEST SAMPLE SIZE = 59
DESIRED PROFITABILITY RATIO = 10
PERCENT PROFIT (OF SELLING PRICE) = 1

CONFIDENCE FOR DESIRED PROFITABILITY RATIO = .9903236

PROFITABILITY CONFIDENCE PROGRAM (PROFCON)

OBSERVED RELIABILTY FROM TEST = .9977175
TEST SAMPLE SIZE = 3
DESIRED PROFITABILITY RATIO = 3.5
PERCENT PROFIT (OF SELLING PRICE) = 1.5

CONFIDENCE FOR DESIRED PROFITABILITY RATIO = .8064986

EXAMPLES OF USING THE "SAMPSIZ" PROGRAM

SAMPLE SIZE PROGRAM (SAMPSIZ)

DESIRED PROFITABILITY RATIO= 2
PERCENT PROFIT (OF SELLING PRICE)= 1.75
FAILURE RATE (ENTROPY) SAFETY FACTOR= 1.9
CONFIDENCE DESIRED FOR PROFITABILITY RATIO= .75

REQUIRED SAMPLE SIZE = 2

SAMPLE SIZE PROGRAM (SAMPSIZ)

DESIRED PROFITABILITY RATIO= 1.75
PERCENT PROFIT (OF SELLING PRICE)= 1
FAILURE RATE (ENTROPY) SAFETY FACTOR= 2
CONFIDENCE DESIRED FOR PROFITABILITY RATIO= .95

REQUIRED SAMPLE SIZE = 11

SAMPLE SIZE PROGRAM (SAMPSIZ)

DESIRED PROFITABILITY RATIO= 2.5
PERCENT PROFIT (OF SELLING PRICE)= 2
FAILURE RATE (ENTROPY) SAFETY FACTOR= 1.4
CONFIDENCE DESIRED FOR PROFITABILITY RATIO= .9

REQUIRED SAMPLE SIZE = 26

CONCLUSION

It can be seen from the examples in using the two computer programs ("PROFCON" and "SAMSIZ") that this mathematical formulation of profit assurance and reliability assurance provides product life testing managers with a powerful and systematic approach to the design and analysis of product durability testing programs. Without such a systematic approach to life testing there is a real danger of just "flying blind" and becoming lost in a hopeless mess of spotty statistics and unsound decisions which could wreak havoc for a manufacturer.