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A HANDY COMPUTER PROGRAM FOR CALCULATING SEMI-PARAMETRIC RANKS TO USE IN CONSTRUCTING CONFIDENCE BANDS FOR WEIBULL PLOTS

INTRODUCTION

As every experienced Weibull Analyst knows, the 90% Confidence Bands derived from classical non-parametric 95% Ranks and 5% Ranks are so wide that nobody could possibly believe that Weibull predictions are all that bad. For this reason we have been forced to find a more reasonable basis for rank tables. Such a basis is to be found in what are called Semi-Parametric Ranks , as explained and applied in this bulletin.

THE QUANTITATIVE BASIS FOR SEMI-PARAMETRIC RANKS

DEFINITION: If a point on a Weibull line is at Median Rank Q, then the **Median Entropy** at that point is

$$\varepsilon_{.50} = -\ln(1 - Q) = \ln[1/(1-Q)] \quad .$$

Furthermore, at the location of Rank Q the non-parametric standard error of the natural logarithm of the Entropy form sample size is

$$_{NP}SIGMA_{\ln \varepsilon} = 1/\sqrt{Q N}$$

On the other hand, if we know the true Weibull slope for sure, then the standard error at the same point is

$$_{P}SIGMA_{\ln \varepsilon} = 1/\sqrt{N} \quad .$$

This is called the Parametric Standard Error of ln(Entropy) at a point.

Now , to formulate the Semi-Parametric Standard Error of the natural logarithm of the Entropy at the point with Median Rank Q we go halfway between **QN and N** in the denominators of the two formulas, i.e., Non-Parametric and Parametric .

Now, $$.5(Q\,N + N) = N(.5 + .5Q)$$

Thus, the Semi-Parametric Standard Error of ln(Entropy) at the point on a Weibull plot with Median rank Q is

$$ {}_{SP}\mathrm{SIGMA}_{\ln \varepsilon} = \frac{1}{\sqrt{N(.5 + .5Q)}} $$

Using this Semi-Parametric Standard Error in a Log Normal Distribution of Entropy we are able to come up with a computer program for Semi-Parametric Ranks.

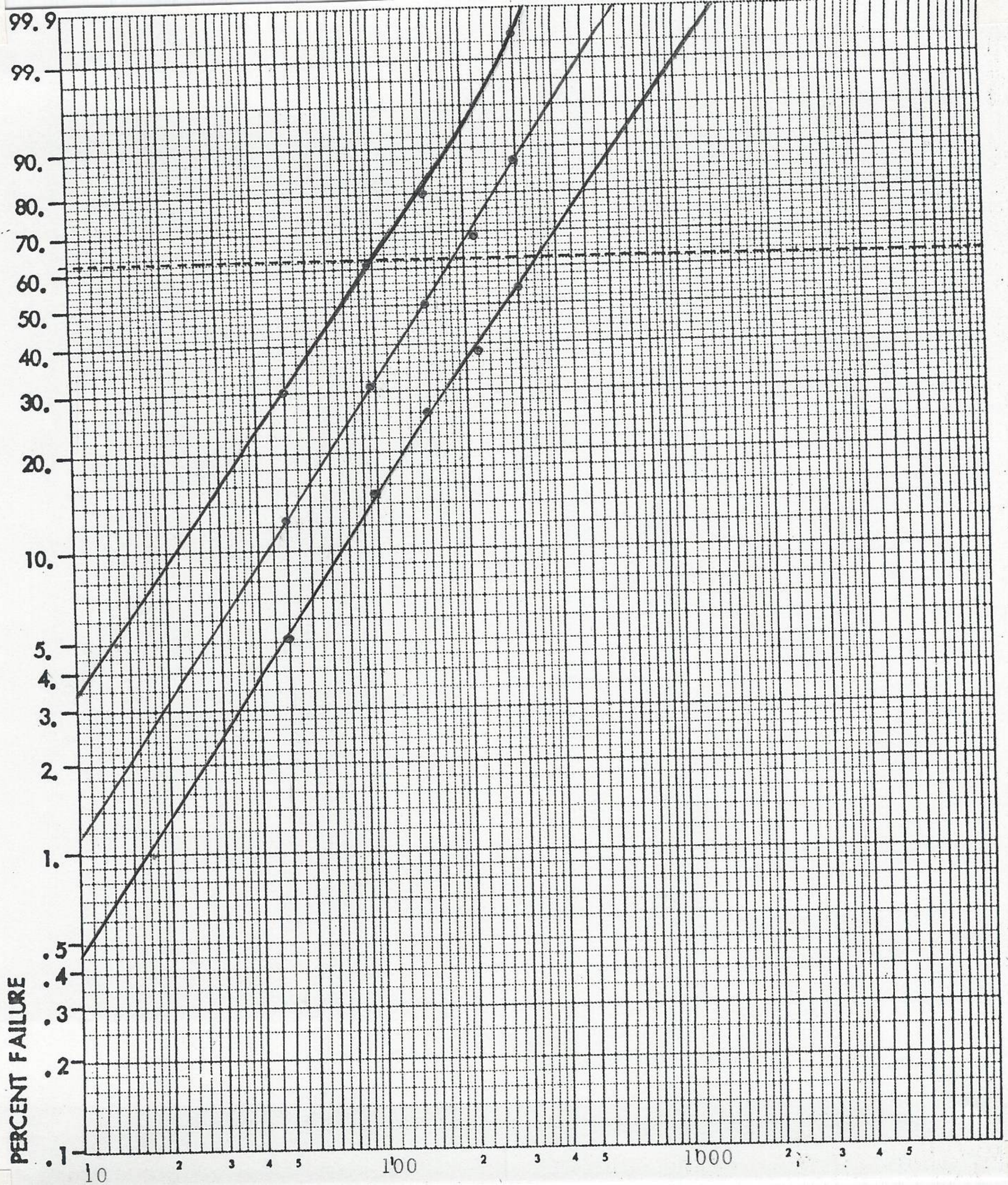
A TYPICAL EXAMPLE

Consider the case of a sample of 5 failures, as follows :

(j) FAILURE NO.	(N = 5) LIFE IN HOURS	(Benard's Formula) (j-.3/N+.4) MEDIAN RANK
1	51	.1296296
2	97	.3148148
3	150	.5000000
4	220	.6851851
5	300	.8703703

The Weibull plot for this data set is the straight line in the middle of Figure 1 .

The 90% Confidence Band boundaries in Figure 1 are obtained from the Semi-Parametric 95% Ranks and 5% Ranks as calculated in the computer program listed on Page 6.



99.9
99.
90.
80.
70.
60.
50.
40.
30.
20.
10.
5.
4.
3.
2.
1.
.5
.4
.3
.2
.1
PERCENT FAILURE
10
2
3
4
5
100
2
3
4
5
1000
2
3
4
5

WEIBULL PLOT OF THE DATA (WITH 90% CONFIDENCE BAND)

(FROM SEMI-PARAMETRIC RANKS)

LIFE (HOURS) ———>

FIGURE 1

```
10 PRINT TAB(20)"SEMI-PARAMETRIC PROGRAM-(SEMPRNK)"
15 PRINT
20 INPUT "CONF. LEVEL OF RANK = ";C
25 PRINT
30 INPUT "SAMPLE SIZE =";N
35 PRINT
40 PRINT "SEMI-PARAMETRIC";100*C;"% RANK TABLE FOR SAMPLE SIZE";N
45 PRINT
50 FOR J = 1 TO N
60 M = (J-.3)/(N+.4)
70 IF M>.5 THEN M = 1 - M
80 W = C/(1-C)
90 V=SQR(N*(.5+.5*M))
100 U = .55/V
110 Y = W^U
120 F = (J-.3)/(N+.4)
130 Z = 1-(1-F)^Y
140 PRINT "SEMI-PARAMETRIC";100*C;"% RANK OF ORDER STATISTIC";J;"=";Z
150 NEXT J
160 END
```

```
                    SEMI-PARAMETRIC PROGRAM-(SEMPRNK)

SEMI-PARAMETRIC 95 % RANK TABLE FOR SAMPLE SIZE 5

SEMI-PARAMETRIC 95 % RANK OF ORDER STATISTIC 1 = .3050608
SEMI-PARAMETRIC 95 % RANK OF ORDER STATISTIC 2 = .6029202
SEMI-PARAMETRIC 95 % RANK OF ORDER STATISTIC 3 = .7980258
SEMI-PARAMETRIC 95 % RANK OF ORDER STATISTIC 4 = .9406056
SEMI-PARAMETRIC 95 % RANK OF ORDER STATISTIC 5 = .9952779
```

```
                    SEMI-PARAMETRIC PROGRAM-(SEMPRNK)

SEMI-PARAMETRIC 5 % RANK TABLE FOR SAMPLE SIZE 5

SEMI-PARAMETRIC 5 % RANK OF ORDER STATISTIC 1 = 5.158669E-02
SEMI-PARAMETRIC 5 % RANK OF ORDER STATISTIC 2 = .1433746
SEMI-PARAMETRIC 5 % RANK OF ORDER STATISTIC 3 = .259445
SEMI-PARAMETRIC 5 % RANK OF ORDER STATISTIC 4 = .3769282
SEMI-PARAMETRIC 5 % RANK OF ORDER STATISTIC 5 = .5413256
```

SPECIAL NOTE ON LOGARITHMIC SYMMETRY (COMMON ENTROPY RATIO)

There is a Common Entropy Ratio at any order statistic point on a Weibull plot and its Semi-Parametric boundaries. Take, for example, the first order statistic in 5. We have

95% Rank	=	.3050608
Median Rank	=	.1296296
5 % Rank	=	.0515867

Entropy at 95% Rank = -ln(1 - .3050608) = -ln(.6949392) = .3639309

Entropy at Median Rank = -ln(1 -.1296296) = -ln(.8703704) = .1388364

Entropy at 5 % Rank = -ln(1 - .0515867) = -ln(.9484133) = .0529649

Thus,

$$\frac{\text{ENTROPY AT 95\% RANK}}{\text{ENTROPY AT MED.RANK}} = \frac{.3639309}{.1388364} = 2.62129$$

and,

$$\frac{\text{ENTROPY AT MED. RANK}}{\text{ENTROPY AT 5\% RANK}} = \frac{.1388364}{.0529649} = 2.62129$$

Thus, we have verified that there is a Common Entropy Ratio at the earliest failure in five. The same fact can be shown for all of the other order statistics.