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THE MATHEMATICAL BASIS FOR REALISTIC SAMPLE SIZES IN SUCCESS RUN TESTING (DERIVATION OF THE COMPRESSED SUCCESS RUN THEOREM)

INTRODUCTION

Test sample sizes for success runs are of primary interest in demonstrating a product's reliability to a specified life target. However, the sample sizes required according to the classical assumption used about the population fraction defective in order to demonstrate reliability with some desired confidence level are so ridiculously large that no industrial manager involved in the testing of expensive specimens or assemblies will ever willing to test that many items, for he knows by his own good common sense that such large sample sizes are a bunch of mathematical garbage. Why is this? That's a good question which needs to be answered to everyone's satisfaction before any success run test programs are designed. It is our intention in this bulletin to derive the compressed success run theorem, which should be used in designing success run tests to specific life targets. By using the compressed success run theorem we arrive at very reasonable sample sizes which are only a small fraction of the ridiculously large sample sizes required by the classical approach.

GEOMETRICAL REPRESENTATION OF THE PROBLEM

The problem we are discussing involves the compressed linear interval containing possible fractions failed at some target life for the product to be manufactured and sold.

Let F_o = Worst Possible Fraction of Items Failed in a Given Target Service Period

Then, the range of possible values for the fraction defective to the target ranges from 0 to F_o . This is pictured geometrically in Figure 1 below:

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I-----------------------------------------------I
0          Fraction Defective --------->        Fo
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FIGURE 1

Let X_o = Testing Target (i.e., the test period to which we get a success run of N consecutive successes.)

<u>QUESTION:</u> What is the population defective at X_o with confidence C of not being exceeded if we obtain N consecutive successes to X_o?

We pictured this situation in Figure 2, where we start out with a prior **Rectangular Distribution** between 0 and F_o. (Worst of N + 1 at Z)

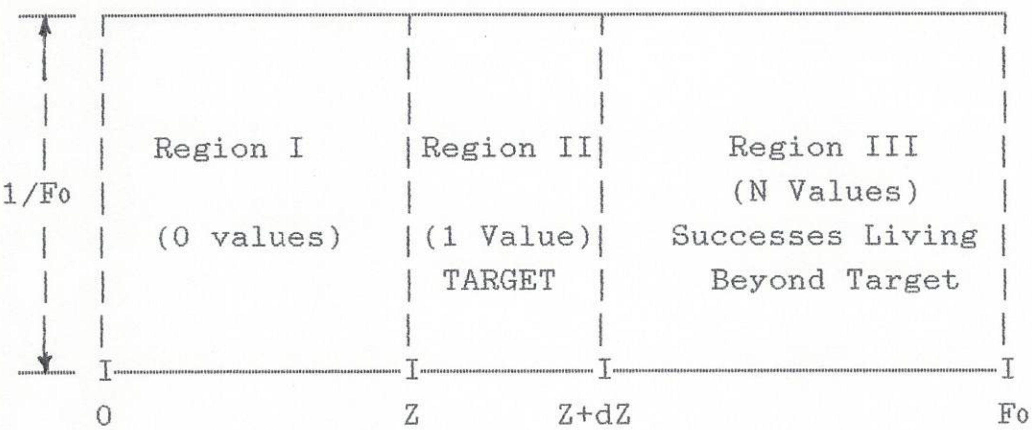

FIGURE 2

Let Z = Fraction Defective at Target X_o.
(Z lies somewhere between 0 and F_o.)

By the **Multinomial Theory of Probability** the probability of having zero items in **Region I**, exactly 1 item in **Region II**, and exactly N items in **Region III** is

$$\frac{(N+1)!}{0!\,1!\,N!}\left[\frac{Z}{F_o}\right]^0\left[\frac{dZ}{F_o}\right]^1\left[\frac{F_o - Z}{F_o}\right]^N = g(Z)dZ\text{ , where}$$

$$g(Z) = \frac{N+1}{F_o}\left[1 - \frac{Z}{F_o}\right]^N$$

is the PROBABILITY DENSITY FUNCTION of Z.

Hence, the **Cumulative Distribution Function** of the fraction defective Z is

$$G(Z) = \int_0^Z \frac{N+1}{Fo} \left[1 - \frac{Z}{Fo} \right]^N dZ = 1 - \left[1 - \frac{Z}{Fo} \right]^{N+1}$$

For confidence C, we must have $1 - (1 - Z/Fo)^{N+1} = C$.

Solving this for Z we obtain $Z = Fo \left[1 - (1 - C)^{1/N+1} \right]$

Now, $Fo = 1 - A$; A = Minimum (worst) Reliability

$Z = 1 - R$; R = Reliability of Population (to target)

$$\text{So , } 1 - \frac{Z}{Fo} = 1 - \frac{1 - R}{1 - A} = \frac{R - A}{1 - A}$$

$$\text{Therefore, } \frac{R - A}{1 - A} = (1 - C)^{1/N+1}$$

From this $R = A + (1 - A)(1 - C)^{1/N+1}$

The last formula from previous page is the **COMPRESSED SUCCESS RUN THEOREM**, which tells us what the **RELIABILITY** is with **CONFIDENCE C** when we have obtained N consecutive successes to a **LIFE TARGET**. So, strictly speaking, we should put a subscript C on the RELIABILITY R, and write the formula as follows:

$$R_C = A + (1 - A)(1 - C)^{1/N+1}$$

NOTE: In case A = 0 this becomes the

CLASSICAL FORMULA $R_C = (1 - C)^{1/N+1}$

COMPARISON OF RESULTS IN VARIOUS SITUATIONS

From the formula $R_c = A + (1 - A)(1 - C)^{1/N+1}$, where

A = Worst possible reliability to test target

N = Success Run

C = Confidence

Rc = Reliability with Confidence C , we obtain the following table of required **SUCCESS RUN SAMPLE SIZES** for various values of the Worst Reliability **A** (i.e., A = 0 , A = .5 , and A = .8)

Let us take C = .90 (90 % Confidence)

DESIRED $R_{.90}$	N (FOR A = 0)	N (FOR A = .50)	N (FOR A = .80)
.95	44	21	8
.99	229	114	44
.999	2301	1150	459
.9999	23,024	11,511	4604

NOTE: Solving $R_c = A + (1 - A)(1 - C)^{1/N+1}$ for the SUCCESS RUN SAMPLE SIZE N , we obtain

$$N = -1 + \frac{\ln(1 - C)}{\ln\left[\frac{R_c - A}{1 - A}\right]}$$

CONCLUSION

The success run sample size N for a Minimum Reliability of **A** is approximated by taking the CLASSICAL SUCCESS RUN SAMPLE SIZE for **ZERO** Minimum Reliability and multiplying it by the factor (1 - A). Thus , in the table on the previous page we see that for A = .5 the sample size is about **HALF** of that for A = 0 , and that for A = .8 the sample size is only about **1/5** **(20%)** of that for A = 0 .

This illustrates the importance of knowing what the worst possible reliability is in a given situation , if we want to avoid excessive testing.