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DETROIT RESEARCH INSTITUTE
P.O. BOX 36504 • GROSSE POINTE, MICHIGAN 48236 • (313) 886-7976

LEONARD G. JOHNSON
EDITOR

WANG H. YEE
DIRECTOR

ESTIMATING A FIELD GOAL LINE FOR ENGINE LIFE IN SERVICE

INTRODUCTION

A TYPICAL ENGINE TESTING SITUATION

Suppose a government standard on a certain aircraft engine requires the engine to survive a 50% overload for 200 hours without any need to interupt its operation. It is reckoned that this would assure 1000 hours of trouble-free service under field conditions. The company with the contract agreement to supply 100 such engines estimates that a catastrophic failure could cost 150 million dollars, while the total profit from 100 sold engines would be 2 million dollars. Furthermore, the company desires nominally one chance in two hundred that an engine would experience operating failure before each 1000 hour inspection point in the field.

SEVEN IMPORTANT QUESTIONS FOR THE SITUATION

QUESTION # 1: What is the field goal line , assuming the engine is "HEALTHY" for 5000 hours, after which it exhibits a Normal distribution of failures ?

QUESTION # 2: If 200 hours represents 50% overload and corresponds to 1000 hours in the field, what is the slope of the "S - N" diagram on log-log paper?

QUESTION # 3: What is the goal line for 50% overload testing?

QUESTION # 4: How many engines should be run trouble-free for 200 hours at 50% overload, if it has been established that under no circumstances could there be more than 10 troubled engines in 1000 hours of operation of our 100 sold engines in the field?

QUESTION # 5: Suppose 4 engines are tested at 50% overload until all fail, with the following results: Failures at 4350 Hrs., 2620 Hrs., 5150 Hrs., and 3680 Hrs.

What is the confidence that these engines indicate compliance to the goal line for the 50% overload?

QUESTION # 6: In order to assure the engine manufacturer of making at least twice as much profit from successful engines as might be lost from failed engines, how high must the odds be made in favor of successfully meeting the field goal, if one failure in twenty is catastrophic?

QUESTION # 7: Five engines are tested at 50% overload with the following results :

Engine # 1	failed at	5005 Hrs.
Engine # 2	unfailed at	6000 Hrs.
Engine # 3	unfailed at	4500 Hrs.
Engine # 4	failed at	7520 Hrs.
Engine # 5	failed at	6510 Hrs.

What is the confidence of meeting the 50% overload goal line?

THE FIELD GOAL LINE

In this bulletin we shall clarify the process of determining the field goal line on Entropy paper when we are told the required hours of trouble-free service life specified in a contract for the engines sold by their supplier. Furthermore, in most cases, a buyer, like the military, will specify that the contractor should be able to run the proposed engine design at some degree of overload (say 50% overload) and have it survive a reduced number of trouble-free hours of operation.

In the situation described in the introduction we are told that :

(1) In service, the required trouble-free hours of operation is 1000 hours.

(2) At 50% overload, the required trouble-free hours of operation is 200 hours.

(3) In Question #1 we are told that the engine design must remain "HEALTHY" for the first 5000 hours in service. What this means is that the Mean Time Between Repairable Failures must be constant, with each trouble-free period between such failures being at least 1000 hours.

(4) There must be only 1 chance in 200 of violating the 1000 hour trouble-free service time.

ANALYSIS

Since we require 1/200 of a failure per engine in 1000 hours of service, this amounts to specifying that

ENTROPY AT 1000 HRS. = 1/200 = .005

Since the engine design is supposed to be "HEATHY" for 5000 hours of service, it must exhibit a Weibull slope of unity (i.e., an exponential distribution) for 5000 hours, with the Entropy function

$\mathcal{E}_1(x) = x/\theta_1$, such that $\mathcal{E}_1(1000) = 1000/\theta_1 = .005$,
which implies $\theta_1 = 200{,}000$ hours
(Characteristic Life for first 5000 hours)

Then, $\mathcal{E}_1(5000 \text{ Hrs.}) = 5000/200{,}000 = .025$, which is the Entropy at 5000 Hrs. of service.

After 5000 Hrs. the engine failures obey a Normal distribution represented by a Weibull slope of 3.5 . Thus , for

$$x > 5000 \text{ Hrs.} : \text{Entropy} = \mathcal{E}_2(x) = (x/\theta_2)^{3.5} ,$$

where θ_2 = Characteristic Life for the Field Goal Line before 5000 hours.

Thus, at 5000 hours the Entropy for the start of the Normal distribution must be the same as it is at 5000 hours where exponential distribution ends. So, we have the equation

$$(5000/\Theta_2)^{3.5} = .025 , \quad \text{from which}$$

$$\Theta_2 = 5000/(.025)^{1/3.5} = 14,345 \text{ Hrs.}$$

CONCLUSION

The field goal line consists of an initial "HEALTHY" slope of unity for 5000 hours on Entropy paper with Θ_1 = 200,000 hours, followed by a wearout slope of 3.5 on Entropy paper beyond 5000 hours with Θ_2 = 14,435 hours. This field goal line is shown on Entropy paper in Figure 1.

NOTE: For each slope , the Characteristic Life is a Unit Entropy.

Subsequent bulletins will discuss the other six questions in the list of Seven Important Questions.

FIGURE 1 : LOG-LOG PLOT FOR FIELD GOAL LINE