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ENTROPY METHOD OF MONITORING ASSEMBLY LINE RELIABILITY

INTRODUCTION

Over the past twenty years the author has found numerous applications for the Entropy concept, wherein Entropy is defined to be the negative logarithm of a probability. In reliability studies the probability of survival of an item or system is of paramount interest. It has been found that the reliability of any system can be conveniently analyzed by using the Entropy approach , since Entropy can be shown to represent the build-up of accumulated system failures (i. e. , breakdowns). As a consequence of this property of Entropy (being the accumulation of breakdowns) it has been possible to analyze electric power transmission systems of public utilities by keeping tab on blackouts in the communities they serve. Other applications have been found in the analysis of field failures of sold motor vehicles, agricultural equipment , appliances, and in fact, any manufactured item subject to failures and their consequent effects on warranty and profitability .

In this bulletin we discuss another interesting application of Entropy , namely, its usefulness in monitoring a manufacturing assembly line by treating the assembly line as a system exhibiting a certain rate of accumulation of defective items coming off the line. From this build-up rate of deffectives per assembly line we can judge in which of three possible conditions an assembly line is .

These three possible conditions are :

I : A deteriorating condition, in which the rate of Entropy growth or build-up of deffective items is increasing .

II: A steady state, in which the build-up rate of deffectives is stabilized to a constant rate .

III: A reliability growth condition, in which the build-up rate of deffective items decreasing (i. e., slowing down with time).

We shall shown by means of Entropy plots how to determine into which of these possible conditions any analyzed assembly line falls.

ESTIMATING DAILY DEFECTIVES FROM INSPECTED SAMPLES

Let T_D = The total number of items produced per day on an assembly line .

Let N_i = The number of items inspected on Day #i

Let D_i = The number of defective items out of the N_i inspected on Day #i .

According to the elementary theory of attribute data analysis in reliability we estimate the average assembly line fraction defective on Day #i to be

$$ {}_iF_{ave} = \frac{D_i + 1}{N_i + 2} . $$

Hence , if the daily total production is T_D, we estimate that the number of defectives for Day $\#_i$ will be a number W_i such that

$$ \frac{W_i + 1}{T_D + 2} = \frac{D_i + 1}{N_i + 2} , $$ from which we derive

the formula for W_i :

$$ W_i = -1 + \left(\frac{D_i + 1}{N_i + 2} \right) (T_D + 2) \qquad (1) $$

This formula (1) for W_i is used to count up the estimated accumulated total defects from the assembly line day after day for any desired period of observation (say 20 to 3o consecutive days) , with each day having a specific inspection sample size N_i and s specific number of defectived D_i in the inspected sample for that day. The daily production T_D is assumed to be fixed, although this could be made to vary daily also .

CONSTRUCTING THE ENTROPY PLOT FOR THE ASSEMBLY LINE

Using Formula (1) for Day #1 in our investigation of an assembly line, we would have

$$W_1 = -1 + \left(\frac{D_1 + 1}{N_2 + 2}\right)(T_D + 2) = \text{Total defects for Day \#1}$$

$$= \text{Entropy of assembly line after 1 day.}$$

Likewise, for Day #2, we would have

$$W_2 = -1 + \left(\frac{D_2 + 1}{N_2 + 2}\right)(T_D + 2) = \text{total defects for Day \#2}$$

Hence, after 2 days of observation, we would have the number of defects per assembly line as

$$W_1 + W_2 = \text{Entropy of assembly line after 2 days.}$$

After 3 days we would have

$$W_1 + W_2 + W_3 = \text{Entropy of assembly line after 3 days,}$$

where $W_3 = -1 + \left(\frac{D_3 + 1}{N_3 + 2}\right)(T_D + 2) = \text{Defects on Day \#3}$

etc. etc. etc.

After k days, we would have

$$W_1 + W_2 + W_3 + \ldots + W_k = \text{Entropy of Assembly Line after } k \text{ days.}$$

In general,

$$W_k = -1 + \left(\frac{D_k + 1}{N_k + 2}\right)(T_D + 2) .$$

Now we can construct on log-log paper the ACCUMULATED DAYS as ABSCISSA , and the ENTROPY of the assembly line for the accumulated days as ORDINATE . The slope at any point of the graph obtained on log-log paper determines the condition of the assembly line.

If the slope is GREATER THAN 1 , then the assembly line is GETTING WORSE as far as accumulating defectives is concerned this is because the defect growth function is apower function of exponent greater than 1 .

If the slope is EQUAL to 1 , then the assembly line defect rate is fixed , since a slope of 1 indicates a LINEAR (uniform) growth of defects versus time.

If the slope is LESS THAN 1 , then the assembly line is showing RELIABILITY GROWTH , or a decreasing rate of defect build-up, since a slope less than 1 on log-log paper represents a power function with a fractional exponent less than 1, which means that the defect count per day is decreasing .

EXAMPLE OF AN ASSEMBLY LINE WITH SAMPLING INSPECTION

As an example to illustrate the monitoring of an assembly line , take the 25 days of inspection data on a factory assembly line as given in Table 1 .

(The daily production is $T_D = 500$ units.)

The table consists of 5 columns :

Column 1 : The day number $= i$

Column 2 : The number of units checked on each day $= N_i$

Column 3 : The number of defectives found each day among units checked $= D_i$

Column 4 : The average estimated daily defectives $= W_i$

$$= -1 + \left(\frac{D_i + 1}{N_i + 2} \right) (T_D + 2) .$$

Column 5 : The accumulated defects day by day $= \sum W_i$ = Entropy .

NOTE : Column 4 and 5 are calculated from the daily inspection data totals of N_i and D_i.

Having completed all the Entropy calculations for the 25 days, we construct a graph of the results on log-log paper (Figure 1) , with the cumulative days as abscissas and the Entropies as ordinates .

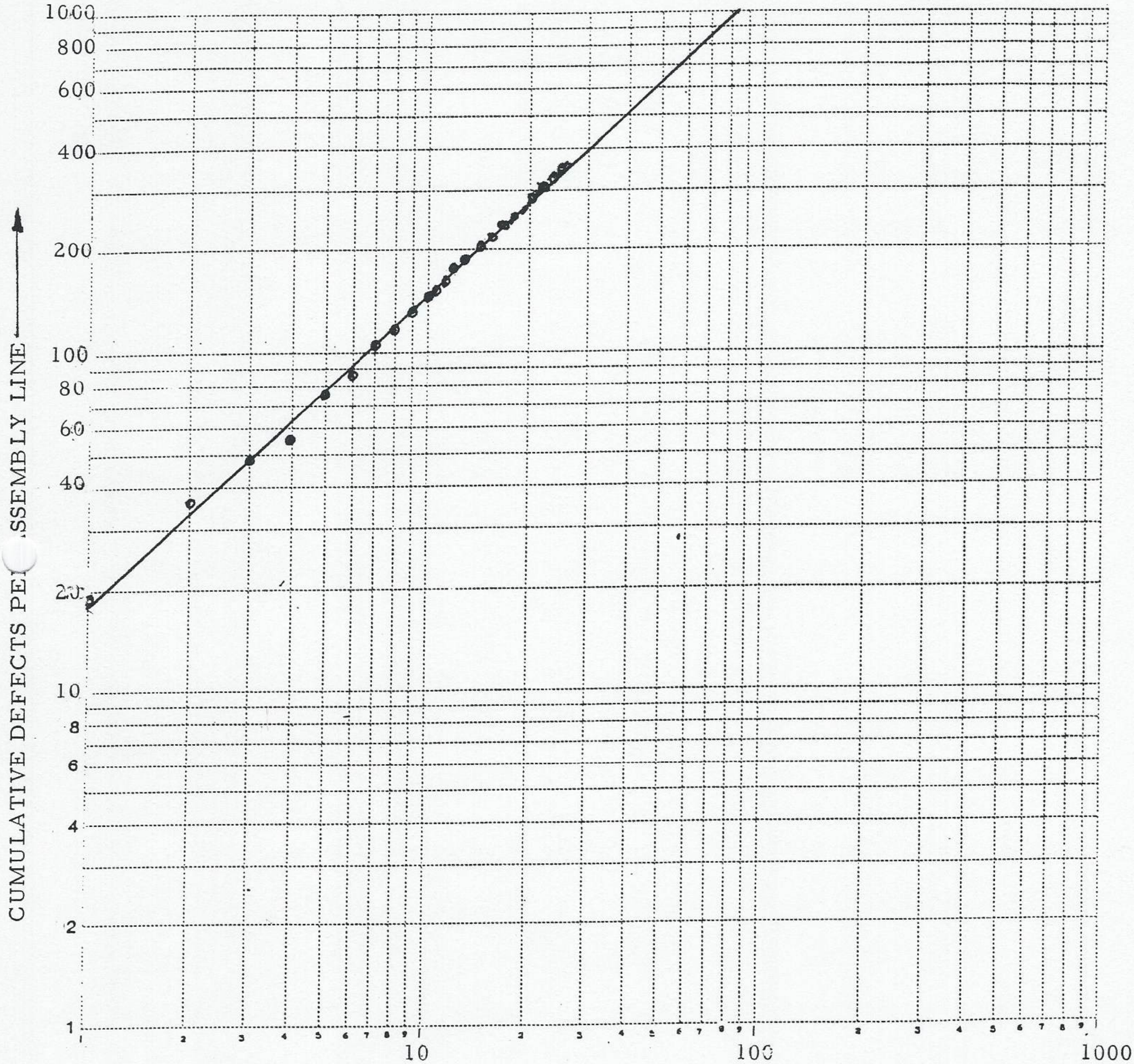
TABLE 1

DAILY FACTORY PRODUCTION = 500 = T_D

DAY NO. i	UNITS CHECKED N_i	NO. DEFECTIVE D_i	AVE. ESTIMATED DAILY DEFECTIVES W_i	CUM. DEFECTS PER FACTORY $\sum W_i$
1	70	2	19.91667	19.91667
2	50	1	18.30769	38.22436
3	90	1	9.91304	48.13740
4	60	0	7.09677	55.23417
5	65	2	21.47761	76.71178
6	45	0	9.68085	86.39263
7	80	2	17.36585	103.75848
8	55	1	16.61404	120.37252
9	60	1	15.19355	135.56607
10	40	0	10.95238	146.51845
11	65	1	13.98508	160.50353
12	85	1	10.54023	171.04376
13	54	0	7.96249	179.00805
14	59	1	15.45902	194.46707
15	68	2	20.51429	214.9816
16	42	0	10.40909	225.39045
17	71	1	12.75342	238.14387
18	80	2	17.36585	255.50972
19	51	1	17.94340	273.45312
20	63	1	14.44615	287.89927
21	79	1	11.39506	299.29433
22	48	1	19.08000	318.37433
23	56	0	7.65517	326.02950
24	73	2	19.08000	345.10950
25	80	1	11.24390	356.35340

LOG-LOG (ENTROPY) PLOT FOR THE FACTORY DEFECT RATE

Weibull slope = .91249 (defect rate is decreasing).



Characteristic time = .041753 day = 20.04 minutes (assuming 8 hr. day)

CUMULATIVE DAYS

FIGURE 1

CONCLUSION

From Figure 1 , representing the graph of the factory data in our illustration, we see that

FIRST , We obtain a straight line graph whose Weibull slope is .91249. Since this is a slope less than unity we conclude that the daily defect rate is decreasing , that is, there is reliability growth or improvement over the 25 days of assembly line experience .

SECOND , We see that the line when projected downward to 1 defect (the first occurrence) predicts such a first occurrence in .041753 day = 20.04 minutes (assuming an 8 hour working day).

It can be seen that this Entropy plotting approach is a good way to check up on the condition of a production line. Had the slope been greater than unity , it would have been an alarm signal telling us that the line was getting worse in daily defect count over the observation period of 25 days. A slope of unity indicates a stable (fixed) defect rate per day , or that the Entropy increases linearly with elapsed time .