Statistical Bulletin
Reliability & Variation Research

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Volume 16
Bulletin 5

October , 1986

THE ENTROPY RATIO DECISION TECHNIQUE USING THE UNIVERSAL LAW OF ODDS

INTRODUCTION

The concept of Entropy (Statistical Weakness) is by far the most useful concept in those decision processes in which we seek Odds or Confidence in some hypothesis. In product assurance the most common questions asked are the following two questions:

QUESTION # 1 : Is the reliability of the proposed product at least as some standard reliability to a standard life target (say 1000 hours) ?

QUESTION # 2 : If we have estimated the life populations of two designs (new design and old design) from test samples , how can we calculate the odds in favor of the new design being at least as good as the old design at a particular life target (say 1000 hours) ?

In this bulletin we shall take examples to illustrate both of these questions and their answers as derived by the Entropy Ratio Technique using the Universal Law of Odds .

THE UNIVERSAL LAW OF ODDS

It is a universal fact of nature that when we seek Odds in favor of superior survivability (i. e., reliability) in a product as compared to a standard (or baseline) we find that regardless of underlying life distribution functions it is always true that

$$\text{ODDS} = (\text{ENTROPY RATIO})^{(\text{ODDS EXPONENT})}$$

This is known as the UNIVERSAL LAW OF ODDS .

Anytime we have an item which has a reliability $R(x_o)$ to a life target x_o we can calculate the Entropy of the item at target x_o as follows :

$$\text{ENTROPY at } x_o = \mathcal{E}(x_o) = \ln \frac{1}{R(x_o)} .$$

So, if we require a Standard Reliability $R_{std.}(x_o)$ to life target x_o we have

$$\text{STANDARD ENTROPY at } x_o = \mathcal{E}_{std.}(x_o) = \ln \frac{1}{R_{std.}(x_o)} .$$

Then, The Entropy Ratio between the Standard at x_o and other item at x_o is

$$\text{ENTROPY RATIO} = \left(\frac{\ln \frac{1}{R_{std.}(x_o)}}{\ln \frac{1}{R(x_o)}} \right) = \frac{\ln \frac{1}{R_{std.}(x_o)}}{\ln \frac{1}{1 - F(x_o)}}$$

$$= \frac{\ln \left[R_{std.}(x_o)\right]}{\ln \left[1 - F(x_o)\right]}$$

Where $F(x)$ = Estimated life CDF of the item tested.

If the sample size of the test item is N_o then the ODDS EXPONENT is given by the formula

$$\text{ODDS EXPONENT} = \frac{\sqrt{N_o}}{0.55} \qquad \left(\text{NOTE : } .55 = \frac{\sqrt{3}}{\pi} \text{ (approx.)} \right)$$

Hence, by the UNIVERSAL LAW OF ODDS, we have

$$\text{ODDS} = \left(\frac{\ln \left[R_{std.}(x_o)\right]}{\ln \left[1 - F(x_o)\right]} \right)^{\frac{\sqrt{N_o}}{.55}}$$

ILLUSTRATION OF A QUESTION OF TYPE # 1

PROBLEM : It is required that a product have a reliability of 90% to a target of 600 hours in service . We test a sample of 10 (all to failure) under service conditions and find the following times to failure (from shortest life to longest life) :

601 hours
1300 hours
1740 hours
2010 hours
2470 hours
2890 hours
3490 hours
4100 hours
4880 hours
6100 hours

What are the Odds that the product will be at least 90% reliable to 600 hours ? In other words, we want to calculate the Odds that at least 90% of all such items sold will last 600 hours in service .

PROCEDURE

We construct an Entropy Plot of the test data as shown on the next two pages.

LIFE	NO. FAILED	NO. ACTIVE	HAZARD	ENTROPY
801 hrs.	1	10	1/10 = .10000	.10000
1300	1	9	1/9 = .11111	.21111
1740	1	8	1/8 = .12500	.33611
2010	1	7	1/7 = .14286	.47897
2470	1	6	1/6 = .16667	.64564
2890	1	5	1/5 = .20000	.84564
3490	1	4	1/4 = .25000	1.09564
4100	1	3	1/3 = .33333	1.42897
4880	1	2	1/2 = .50000	1.92897
6100	1	1	1/1 = 1.00000	2.92897

We plot Life as Abscissa on log-log paper and Entropy as Ordinate on log-log paper , and obtain the Entropy plot shown in Figure 1 .

From the plot we see that the test item Entropy at the standard 600 hours is .06.

The Desired Standard Entropy to 600 hours is

$$\ln \frac{1}{R_{std.}(600\text{ hrs.})} = \ln \frac{1}{.9} = .10536 \quad .$$

Hence , the ENTROPY RATIO is .10536/.06 = 1.756 .

For sample size N_o = 10 , the Odds Exponent is $\sqrt{10}/.55$ = 5.7496

Hence , by the Universal Law of Odds , we get

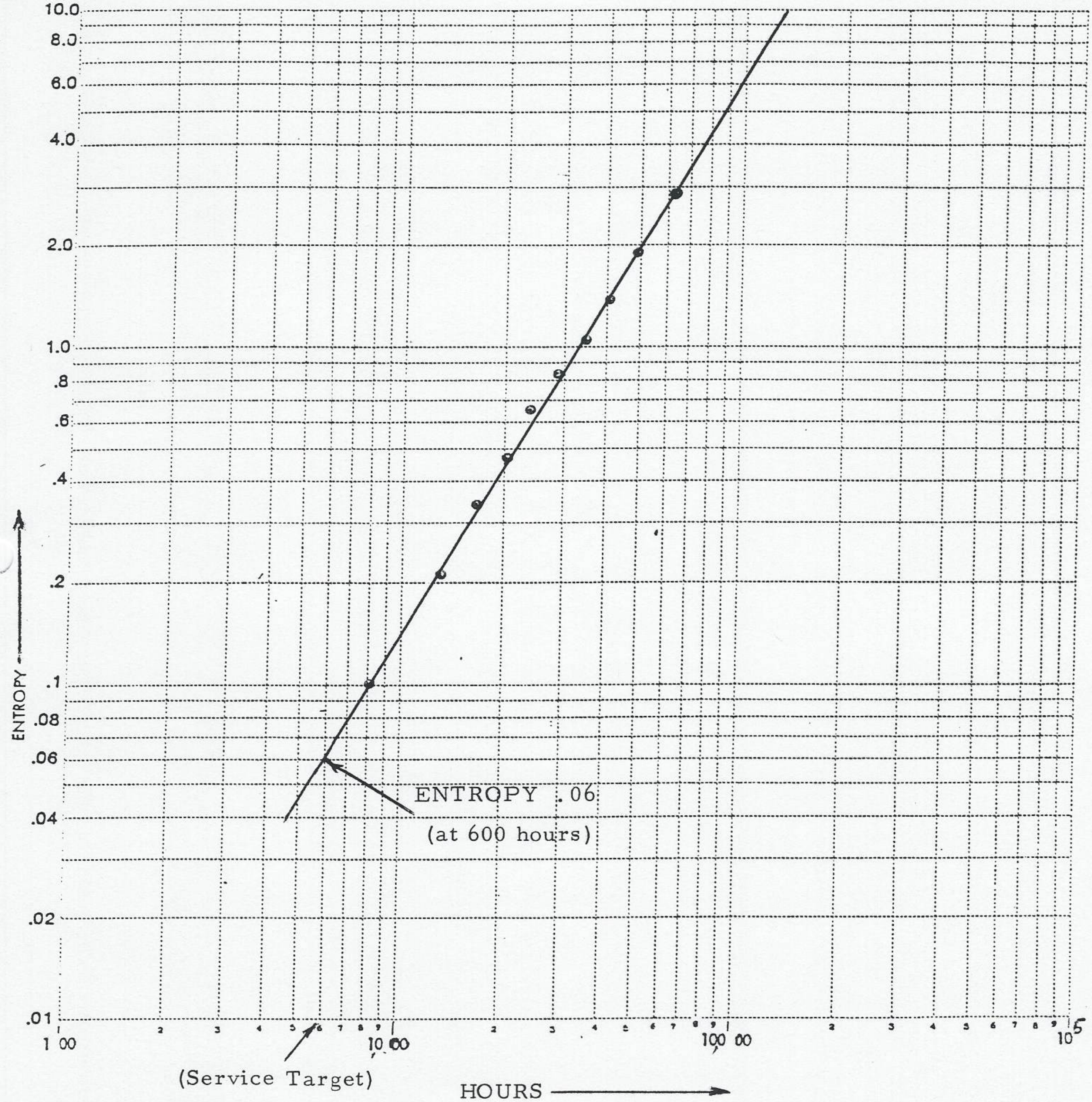
$$\text{ODDS} = (\text{ENTROPY RATIO})^{(\text{ODDS EXPONENT})} = (1.756)^{5.7496} = 25.46 \quad .$$

Therefore , the Confidence that at least 90% of the sold items will last 600 hours in service is

$$\text{CONFIDENCE} = \frac{\text{ODDS}}{1 + \text{ODDS}} = \frac{25.46}{26.46} = .96 \quad .$$

Thus , we are 96% confident that the product will be at least 90% reliable to 600 hours .

LOG - LOG GRID FOR ENTROPY vs. HOURS



N_o = SAMPLE SIZE = 10

FIGURE 1

COMPARING TWO SAMPLE PLOTS AT A TARGET x_o

The general method of comparing two sample plots at a target x_o can be outlined as follows :

Let the two sample plots have estimated CDF's $F_1(x)$ and $F_2(x)$, respectively, where $F_2(x_o) < F_1(x_o)$.

Let N_1 = Sample Size of Plot #1 at x_o

Let N_2 = Sample Size of Plot #2 at x_o

Then the Odds in Favor of #2 being better than #1 at x_o are given by the Universal Law of Odds

$$\text{ODDS} = (\text{ENTROPY RATIO})^{(\text{ODDS EXPONENT})}$$

Where

$$\text{ENTROPY RATIO} = \left(\frac{\ln \dfrac{1}{1 - F_1(x_o)}}{\ln \dfrac{1}{1 - F_2(x_o)}} \right)$$

and

$$\text{ODDS EXPONENT} = \Omega = \frac{k}{.55\left(1/\sqrt{N_1} + 1/\sqrt{N_2}\right)}$$

where

$$k = \sqrt{1 + \frac{\sqrt{N_1 N_2}}{\frac{1}{2}(N_1 + N_2)}}$$

EXAMPLE OF A QUESTION OF TYPE # 2

Suppose Design I has 4 items tested to failure with Weibull parameters $b = 1.8$ and $\theta = 850$ hours, and suppose Design II has 5 items tested to failure with Weibull parameters $b = 2.0$ and $\theta = 1250$ hours. (The Entropy Plots for these samples are shown in Figure 2).

Calculate the Odds that Design II is better than Design I for a service life $x_o = 300$ hours.

SOLUTION

$$F_1(x_o) = 1 - e^{-(x_o/850)^{1.8}} = 1 - e^{-(300/850)^{1.8}} = .14223$$

$$F_2(x_o) = 1 - e^{-(x_o/1250)^{2}} = 1 - e^{-(300/1250)^{2}} = .05597$$

$$\text{ENTROPY RATIO} = \left(\frac{\ln \frac{1}{1 - .14223}}{\ln \frac{1}{1 - .05597}} \right) = \frac{.15342}{.05548} = 2.7653$$

$$\text{ODDS EXPONENT} = \frac{\sqrt{1 + \frac{\sqrt{N_1 N_2}}{1/2(N_1 + N_2)}}}{.55(1/\sqrt{N_1} + 1/\sqrt{N_2})} = \frac{\sqrt{1 + \frac{\sqrt{20}}{4.5}}}{.55(1/\sqrt{4} + 1/\sqrt{5})} = \frac{1.41202}{.52097} = 2.7118$$

$$\text{ODDS} = (\text{ENTROPY RATIO})^{(\text{ODDS EXPONENT})} = (2.7653)^{2.7118} = 15.773$$

$$\text{CONFIDENCE} = \frac{\text{ODDS}}{1 + \text{ODDS}} = \frac{15.773}{16.773} = .94$$

Thus, we are 94% confident that Design II is better than Design I to a service life of 300 hours.

CONCLUSION

From the examples discussed in this bulletin we see that the Universal Law of Odds involving Entropy Ratios is a very convenient tool in making decisions about product reliability from test data when compared to a standard requirement or when compared to another set of test results from another design.

LOG - LOG GRID FOR ENTROPY vs. HOURS

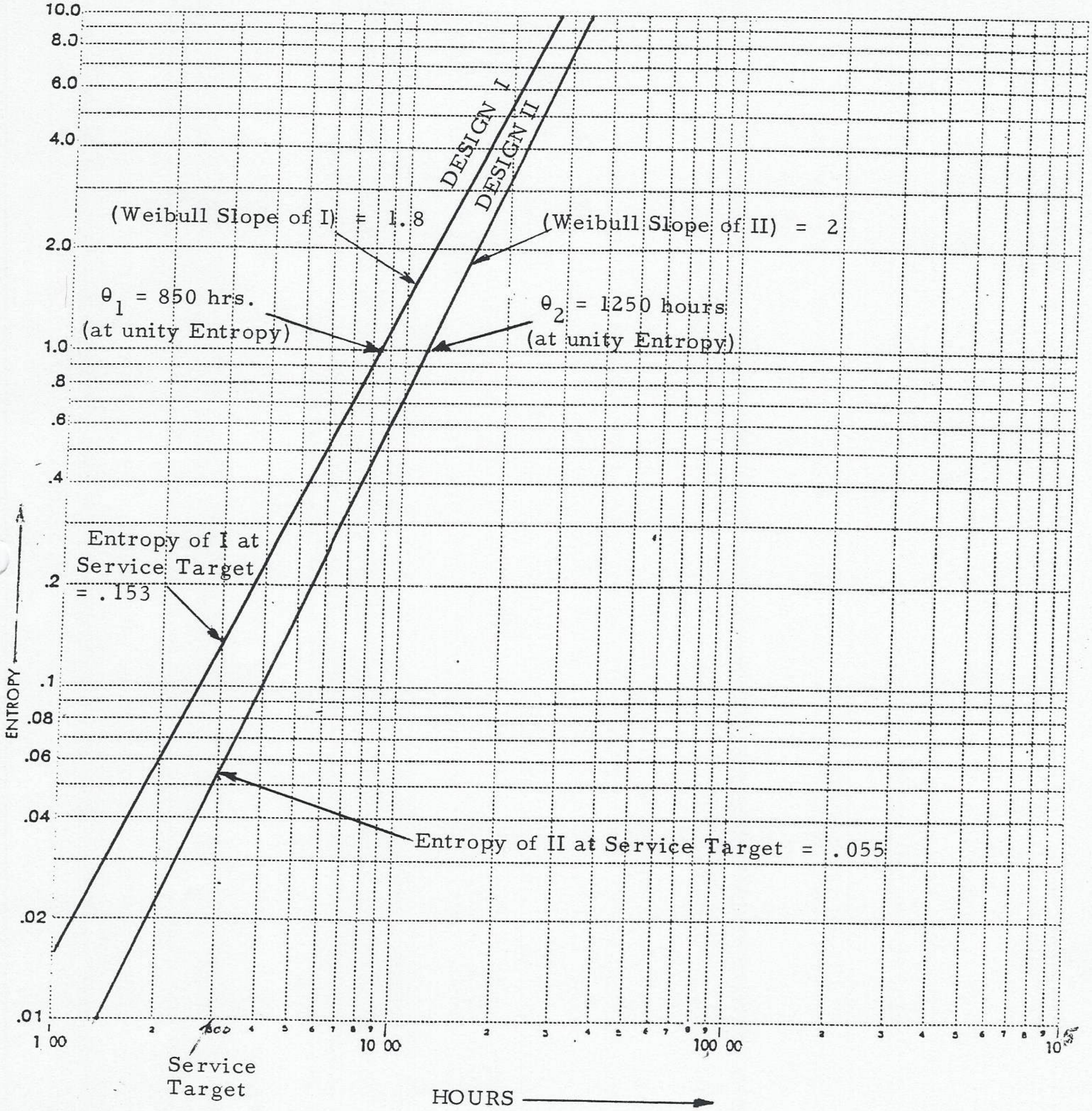


FIGURE 2