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HOW REDUCED LEAD TIME REDUCES PROFITABILITY BY CUTTING SHORT REQUIRED TESTS BY EVIDENCE OF RELIABILITY

INTRODUCTION

A common condition present in product development programs for new designs is a lack of understanding of the logical approach to establishing sufficient lead time in order to gather the evidence required about the product's reliability . In this bulletin we discuss the mathematical approach to evaluating the effects of shortening required lead times , i.e., the effects of settling for only partial evidence instead of the fully required evidence of a product's reliability . In particular , this means that management , being unaware of the significance of the required confidence in product testing programs , often will arbitrarily cut short the number of tests needed , and thus put a pre-mature product on the market , which causes a greater loss than all of the benefits anticipated through earlier introduction to the public .

QUANTIFYING LEAD TIME VERSUS EVIDENCE

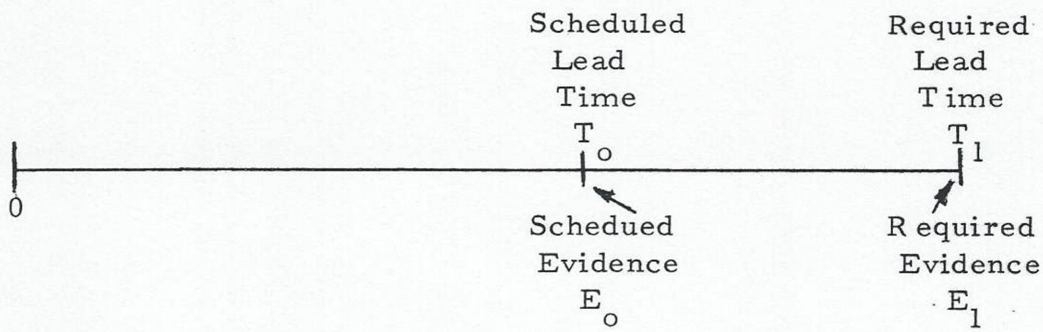


FIGURE 1

Figure 1 schematically depicts the general proportionality relationship for sequential testing, i. e.,

$$\frac{T_o}{E_o} = \frac{T_1}{E_1}$$

CRITICAL QUESTION: At what scheduled lead time T_o does profitability vanish ?

SOLUTION TO THE CRITICAL QUESTION

$$\text{Required Evidence} = E_1 = \ln\left(\frac{C_1}{1 - C_1}\right)$$

Where C_1 = The confidence required for a desired profitability factor K_1,

where $K_1 = \dfrac{G\,C_1}{L(1 - C_1)}$, in which

G = Dollar Gain if in compliance with reliability goal, and

L = Dollar Loss if NOT in compliance with reliability goal.

Profitability vanishes when the profitability factor at scheduled lead time T_o becomes $K_o = 1$, i.e., when the confidence of complying with the reliability goal is reduced to C_o, where

$$\frac{G C_o}{L(1 - C_o)} = 1 \quad , \quad \text{i.e.,} \quad C_o = \frac{L}{L + G} \quad .$$

Then the Evidence at scheduled lead time T_o is reduced to

$$E_o = \ln(C_o/1 - C_o) = \ln(L/G) \quad .$$

Now,

$$E_1 = \ln(C_1/1 - C_1) \quad , \quad \text{where} \quad C_1 = \frac{K_1 L}{K_1 L + G}$$

$$\therefore \quad E_1 = \ln\left(\frac{K_1 L}{G}\right)$$

Therefore, from the proportionality relation for sequential testing, i.e., $T_o/E_o = T_1/E_1$, we see that the formula for T_o is

$$T_o = (E_o/E_1)\, T_1 \quad .$$

Since $E_o = \ln(L/G)$ and $E_1 = \ln(K_1 L/G) = \ln K_1 + \ln(L/G)$,

it follows that

$$T_o = \left[\frac{\ln(L/G)}{\ln K_1 + \ln(L/G)}\right] T_1 = \frac{T_1}{1 + \ln K_1/\ln(L/G)}$$

This formula for T_o represents a lead time so shortened as to wipe out profitability.

A NUMERICAL EXAMPLE

Suppose a product yields one million dollars of profit in case it is complying with a promised reliability , but produces a loss of ten million dollars in case it does not comply with the promised reliability .

Suppose the lead time required was 2 years in order guarantee long run gains three times as large as long run losses . If management wants a shorter lead time than 2 years , at what shortened lead time would profits be completely wiped out ?

SOLUTION

We must evaluate the formula

$$T_o = \frac{T_1}{1 + \ln K_1 / \ln (L/G)}$$

Where T_1 = 2 years (required lead time)

K_1 = 3 (original desire profitability factor , i. e. , log run gains three times long run losses)

L = \$10, 000, 000 (loss if not complying)

G = \$1, 000, 000 (gain if complying)

$$T_o = \frac{2}{1 + \dfrac{\ln 3}{\ln\left(\dfrac{10,000,000}{1,000,000}\right)}} = \frac{2}{1 + \dfrac{\ln 3}{\ln 10}} = \frac{2}{1.47712}$$

$$= 1.354 \text{ years} = 16.25 \text{ months.}$$

Thus , if management reduces the required lead time of 24 months to 16. 25 months or less , all profits will be wiped out .

GENERAL QUESTION REGARDING LEAD TIME REDUCTIONS

QUESTION : If the original lead time T_1 (with profitability factor K_1) is reduced to a shorter lead time T_1' , what is the reduced profitability factor K_1' ?

SOLUTION

The reduced Evidence is $E_1' = (T_1'/T_1)\, E_1$

So, the reduced Confidence C_1' is

$$C_1' = \frac{1}{1 + e^{-E_1'}} = \frac{1}{1 + e^{-(T_1'/T_1)\, E_1}}$$

But , $E_1 = \ln (C_1/1 - C_1)$

So, $$e^{-(T_1'/T_1)\, E_1} = \left(e^{E_1}\right)^{-(T_1'/T_1)} = \left(\frac{C_1}{1 - C_1}\right)^{-(T_1'/T_1)} = (K_1 L/G)^{-(T_1'/T_1)}$$

$$\therefore\ C_1' = \frac{1}{1 + \left(\frac{C_1}{1 - C_1}\right)^{-(T_1'/T_1)}} = \frac{1}{1 + \left(\frac{K_1 L}{G}\right)^{-(T_1'/T_1)}}$$

So , $$1 - C_1' = \frac{\left(\frac{K_1 L}{G}\right)^{-(T_1'/T_1)}}{1 + \left(\frac{K_1 L}{G}\right)^{-(T_1'/T_1)}}$$ and $$\frac{C_1'}{1 - C_1'} = \left(\frac{K_1 L}{G}\right)^{(T_1'/T_1)}$$

K_1' is defined in terms of C_1' as follows :

$$K_1' = \left(\frac{G}{L}\right)\left(\frac{C_1'}{1 - C_1'}\right) = \left(\frac{G}{L}\right)\left(\frac{K_1 L}{G}\right)^{(T_1'/T_1)} = \frac{K_1^{(T_1'/T_1)}}{\left(\frac{L}{G}\right)^{(1 - T_1'/T_1)}}$$

APPLYING THE GENERAL FORMULA TO THE NUMERICAL EXAMPLE

In the numerical example we had

$$K_1 = 3$$

$$T_1 = 24 \text{ Months}$$

$$G = \$1,000,000$$

$$L = \$10,000,000$$

Suppose management wants to reduce the 24 month lead time to 18 months . Then the profitability factor is reduced to

$$K_1' = \frac{K_1^{(18/24)}}{\frac{L}{G}^{(1 - 18/24)}}$$

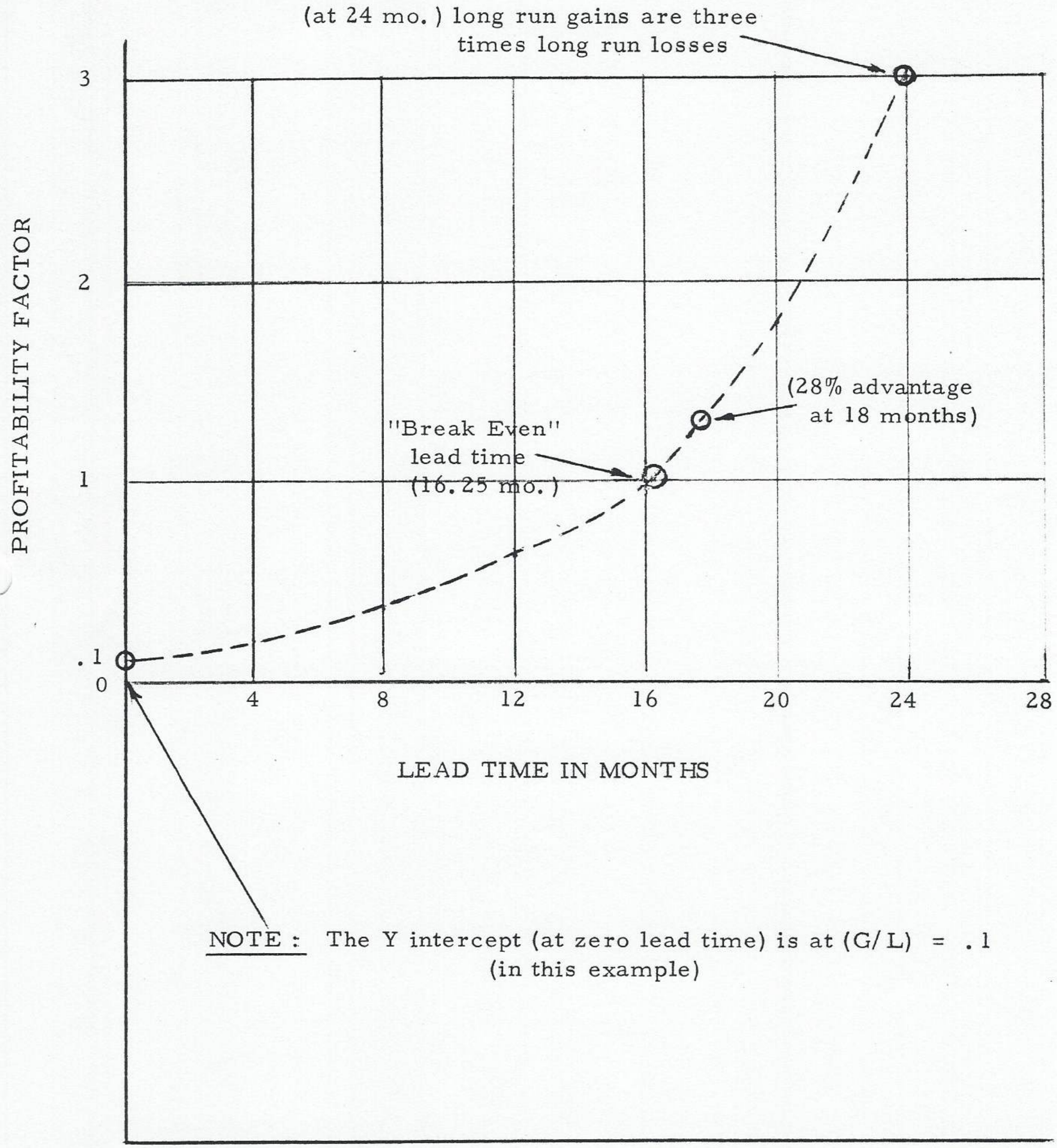
or

$$K_1' = \frac{3^{(3/4)}}{10^{(1/4)}} = \frac{2.27951}{1.77828} = 1.28186$$

Thus , reducing the lead time to 18 months instead of 24 months will reduce profitability so much that we only gain 28% more in the long run than the losses suffered from non-compliance . This is in contrast to gaining 3 times as much as the losses in case the 24 month lead time requirement was allowed to remain .

NOTE : The complete profitability graph for this example appears in Figure 2 .

PROFITABILITY GRAPH FOR THE EXAMPLE



NOTE : The Y intercept (at zero lead time) is at (G/L) = .1 (in this example)

FIGURE 2

CONCLUSION

From the illustrated situations presented in this bulletin we have shown the following :

(a) That it is possible to establish the "Break Even" lead time, which defines the point on the lead time axis below which no profits can be expected .

(b) That whatever lead time is settled for , we can predict the product's profitability from the Evidence gathered within that lead time .

(c) That the reduced profitability factor is equal to

$$\frac{(\text{Original Profitability Factor})^{(\text{Time Ratio})}}{(\text{Money Ratio})^{(1\ -\ \text{Time Ratio})}}$$

Where Money Ratio $=$ (L/G)

$$\text{Time Ratio} = \left(\frac{\text{Shortened Lead Time}}{\text{Original Lead Time}}\right)$$