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THE DOLLAR BASIS OF CONFIDENCE

INTRODUCTION

In our present cost conscious age it is of utmost importance that we know how to relate testing programs to the ultimate profitability of a product which must meet a specified reliability goal in order to perform well enough to produce sufficient custom satisfaction.. This means that we can no longer be arbitrary about how much testing is required , i.e., we cannot just assume some level of confidence (say 90%) and conclude this confidence level to be sufficient to produce a desired profit. This is because confidence has a dollar basis which depends on the magnitude of the dollars gained when we are in compliance with a specified reliability promise as well as the magnitude of the dollar loss in case we should fail to comply with the promised reliability. Our purpose in this report is to clarify the precise dollar basis of confidence in terms of profitability and dollar gains and losses due to the performance of our product.

QUESTION: What is the appropriate confidence level C ?

BASIC RELATION

(Dollar Gain from being Correct) C = K (Dollar Loss from being Wrong) (1 - C)

K = Profitability Factor

$$C = \frac{K(\text{Dollar Loss Wrong})}{(\text{Dollar Gain Correct}) + K(\text{Dollar Loss Wrong})}$$

$$C = \frac{K}{\left(\frac{\text{Dollar Gain Correct}}{\text{Dollar Loss Wrong}}\right) + K}$$

SUPPOSE : $\frac{\text{Dollar Gain}}{\text{Dollar Loss}} = \frac{1}{100}$ (for K = 1 :)

$C = \frac{1}{.01 + 1}$ .99 "FAIR" game

The Sample Size required depends on the Confidence Level to be attained.

The confidence level to be attained is made up of

(1) the prior confidence

(2) the sample confidence

The criterion for selecting the confidence level to be attained is built up by deciding how large a ratio is desired for

$$\left(\frac{\text{Long Run Expected Dollar Gains}}{\text{Long Run Expected Dollar Losses}}\right).$$

Long Run Expected Dollar Gains

= (Dollar Gain per Good Event) x (Confidence of a Good Event)

Long Run Expected Dollar Losses

= (Dollar Loss per Bad Event) x (Confidence of a Bad Event)

NOTE : A "Good Event" is a situation in which we meet the reliability goal . A "Bad Event " is a situation in which we fail to meet the reliability goal .

In the field of gambling we call a game "Fair" if

Long Run Expected Dollar Gains per Contestant

= Long Run Expected Dollar Losses per Contestant .

In consumer product business the manufacturer should see to it that

$$\frac{\text{Long Run Expected Dollar Gains}}{\text{Long Run Expected Dollar Losses}} = K > 1$$

(The selection of factor K is the manufacturer's own choice.)

If C = Confidence of a Good Event

Then 1 - C = Confidence of a Bad Event

$$(\text{Dollar Gain per Good Event})\ C = K\ (\text{Dollar Loss per Bad Event})\ (1 - C)$$

Solve this for C :

$$C = \frac{K\ (\text{Dollar Loss per Bad Event})}{(\text{Dollar Gain per Good Event}) + K\ (\text{Dollar Loss per Bad Event})}$$

EXAMPLE

A certain manufacturer's product will yield a gain (profit) of 10 million dollars when it performs as advertised.

However , should it fail to perform as advertised , the predicted losses due to litigation and customer dissatisfaction is 100 million dollars .

The manufacturer wants to see long run expected gains to be 10 times as large as long run expected losses .

The proper confidence level is

$$C = \frac{10\ (\$100,000,000)}{\$10,000,000 + 10\ (100,000,000)} = .99099 \quad \text{or}$$

99.1% confidence

Must have 99.1% confidence of keeping his promise about the product's performance .

Suppose we settled for 90% confidence of a Good Event.

Long Run Expected Dollar Gains = \$10,000,000 x .9 = \$9,000,000

Long Run Expected Dollar Losses = \$100,000,000 x .1 = \$1,000,000

90% confidence represents 9/1 odds ratio

$$\frac{90\%\ \text{For}}{10\%\ \text{Against}} = \frac{9}{1}$$

Just to <u>break</u> <u>even</u> requires an Odds Ratio $= \dfrac{100,000,000}{10,000,000} = \dfrac{10}{1}$

Odds 10/1 implies C = 10/11 = 10/10 + 1 = 90.91% confidence .

$$\frac{\text{Dollar Loss per Bad Event}}{\text{Dollar Gain per Good Event}} = \frac{L}{G}$$

K = Profitability Factor (wants Gains to be K times Losses)

$$\text{Required Odds} = K\left(\frac{L}{G}\right)$$

$$\text{Required Confidence Level in Testing} = \frac{K\left(\frac{L}{G}\right)}{1 + K\left(\frac{L}{G}\right)}$$

$$= \frac{KL}{G + KL}$$

ANOTHER EXAMPLE

\$250,000,000 (L) loss for unkept promise

\$2,000,000 (G) gain for kept promise

Want to gain twice as much as we ever lose . (K = 2)

Required Odd = K (L/G) = 2 x 250/2 = 250/1

Required Confidence = 250/251 = .996

CONCLUSION

From our discussion (with specific examples) we conclude that the Confidence Required in a testing program depends on Three Main Factors .

These are

I : The dollars which would be lost in case we fail to keep a reliability promise . (this is the quantity L)

II: The dollars gained when our product lives up to the promised reliability. (this is the quantity G)

III : The profitability factor desired as expressed by the ratio

$$K = \frac{G \times (\text{Confidence of Meeting the Goal})}{L \times (\text{Confidence of not meeting the goal})}$$

If we let C = Confidence of meeting the goal

Then 1 - C = Confidence of not meeting the goal

So, $K = \dfrac{GC}{L(1 - C)}$, From which we obtain our

Required Confidence Formula , i.e. ,

$$C = \frac{KL}{G + KL}$$