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THE ENTROPY METHOD OF ACCUMULATING EVIDENCE OF MEETING A RELIABILITY GOAL BY VARIOUS TYPES OF TESTS

INTRODUCTION

In engineering testing programs we come across various types of tests , including

(a) Components Tests

(b) Assembly Tests

(c) Prototype Tests

(d) Proving Ground Tests

(e) Accelerated Lab. Tests

The most frequent question asked about such testing programs is the following :

QUESTION : How can we combine all of the results we obtain from these tests into a composite index of confidence that we are meeting the reliability goal for the product being tested ?

THE NECESSARY FACTORS IN THE STUDY

There are certain necessary factors which must be available if we are to determine a composite confidence index of meeting reliability goals . These are :

I : We must know the Field Goal Line .

II : We must know how the Goal Line changes for each test condition different from field conditions .

III : We must know how to put together component reliabilities into assembly reliabilities .

I : THE FIELD GOAL LINE

The Field Goal Line is simply a line (or curve) on Weibull paper which describes a satisfactory product life in the hands of the customer . For example , in Figure 1 we show a Field Goal Line for an automotive muffler with a B_{10} life of 20,000 miles and a Weibull slope of 3.5 . In order to be at least as good as this Goal Line a data plot of field failures of mufflers must show a Weibull line to the right of this Field Goal Line .

II : GOAL LINE CHANGES WITH TEST TYPE

Anytime we test in a fashion not actually in the field we must know how the Field Goal Line is shifted to agree with the actual test conditions . For example , if life varies inversely as the m^{th} power of stress (or other severity factors) , we can state that

$$\text{LIFE} = \frac{\text{constant}}{(\text{stress})^m} \tag{1}$$

Now suppose, for example , that $m = 4$, and that a certain test increases the stress 20% above the field condition stress . Then the Life Conversion Factor for reducing the Goal Line to the actual test conditions would be the

Conversion Factor $= (1.2)^4 = 2.0736$, i.e.

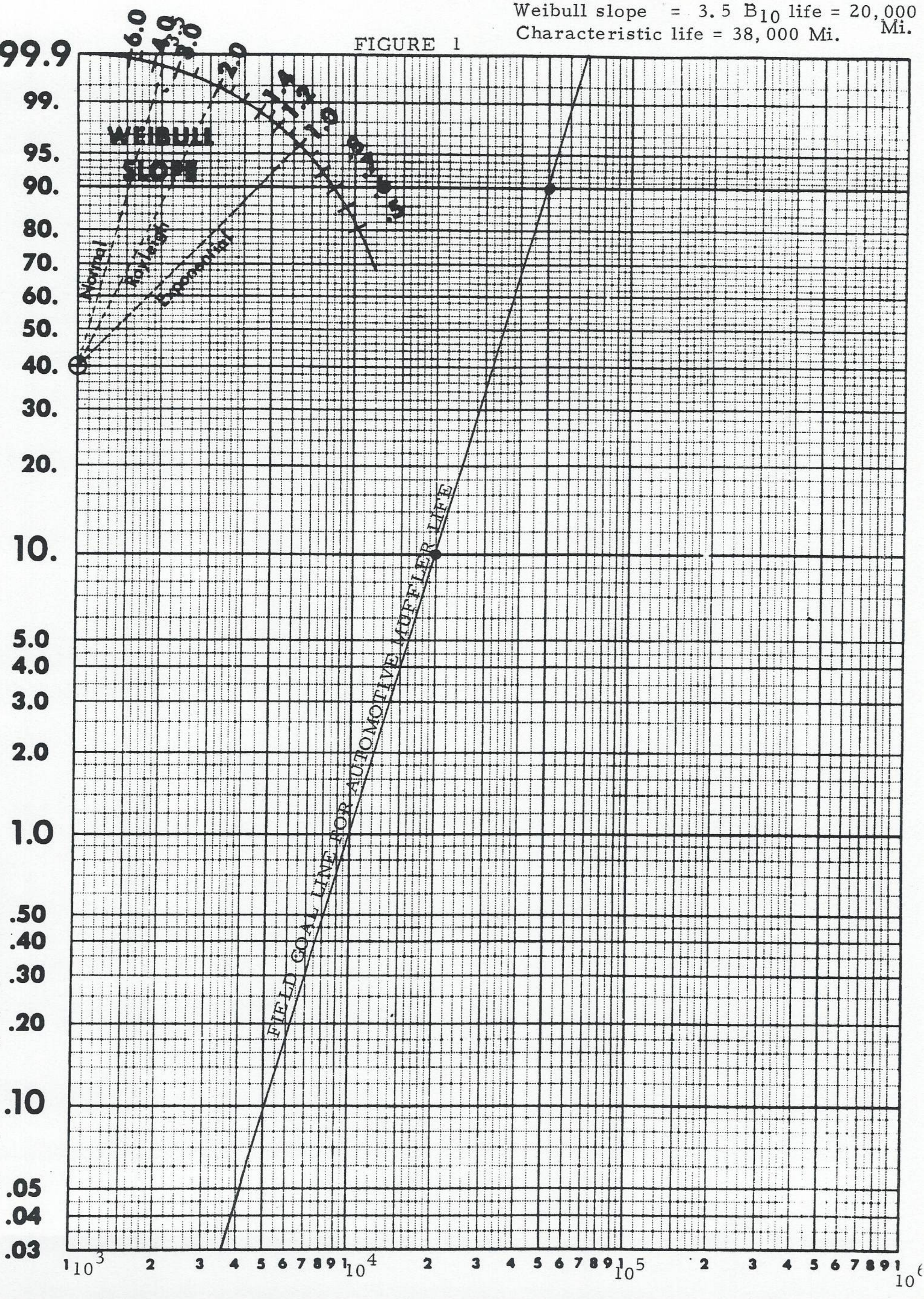
$$\text{TEST LIFE} = \frac{\text{FIELD LIFE}}{2.0736} \tag{2}$$

Thus , the Goal Line for this test on Weibull paper would show life values which are equal to

$$\frac{\text{Field Goal Values}}{2.0736}$$

Weibull slope = 3.5 B_{10} life = 20,000 Mi.
Characteristic life = 38,000 Mi.

FIGURE 1



THE ENTROPY METHOD OF EVALUATION

In the evaluationg test data with respect to a Goal Line on Weibull paper we take the actual test data points at $x_1, x_2, x_3, \ldots, x_N$ and calculate the Entropy Total as follows :

$$\mathcal{E}_{total} = \left(\frac{x_1}{\theta_{test\ goal}}\right)^{b_{test\ goal}} + \left(\frac{x_2}{\theta_{test\ goal}}\right)^{b_{test\ goal}} + \left(\frac{x_3}{\theta_{test\ goal}}\right)^{b_{test\ goal}} + \ldots + \left(\frac{x_N}{\theta_{test\ goal}}\right)^{b_{test\ goal}} \qquad (3)$$

Where

$b_{test\ goal}$ = Test Goal Line Weibull slope

$\theta_{test\ goal}$ = Test Goal Characteristic Life

In case r out of the N test points $(x_1, x_2, x_3, \ldots, x_N)$ are failures , then divide the Entropy Total by r to obtain the AVERAGE ENTROPY PER FAILURE

$$\mathcal{E}_{ave.} = \frac{\mathcal{E}_{total}}{r} \qquad (r \geq 1) \qquad (4)$$

[In case r = 0 (no failures), simply use $\mathcal{E}_{total}$]

HOW TO CALCULATE EVIDENCE

EVIDENCE , In a mathematical sense , is defined as follows :

$$\text{EVIDENCE} = \ln (c/1 - c) \tag{5}$$

Where c = Confidence of meeting the goal

$1 - c$ = Confidence against meeting the goal

so, the ratio $(c/1 - c)$ = Odds in Favor of meeting the goal .

Thus , we can state that $\text{EVIDENCE} = \ln(\text{ODDS})$ (6)

In case we have several tests (say k of them) with failures, we can calculate the average Entropy per failure in each of the k tests as follows :

$$_1\mathcal{E}_{ave.} = \frac{_1\mathcal{E}_{total}}{r_1} \quad (r_1 \text{ failed in Test } 1)$$

$$_2\mathcal{E}_{ave.} = \frac{_2\mathcal{E}_{total}}{r_2} \quad (r_2 \text{ failed in Test } 2)$$

$$\vdots$$

$$_k\mathcal{E}_{ave.} = \frac{_k\mathcal{E}_{total}}{r_k} \quad (r_k \text{ failed in Test } k)$$

To calculate the EVIDENCE from the first test , evaluate the formula

$$E_1 = \frac{\pi}{\sqrt{3}} \sqrt{r_1} \, (_1\mathcal{E}_{ave.} - 1) \quad \left(\text{First Test's Evidence}\right)$$

Likewise , from test # 2 :

$$E_2 = \frac{\pi}{\sqrt{3}} \sqrt{r_2} \, (_2\mathcal{E}_{ave.} - 1) \quad \left(\text{Second Test's Evidence}\right)$$

$$\vdots$$

Finally , from test # k :

$$E_k = \frac{\pi}{\sqrt{3}} \sqrt{r_k} \, (_k\mathcal{E}_{ave.} - 1) \quad \left(k^{\underline{th}} \text{ Test's Evidence}\right)$$

Then , the TOTAL EVIDENCE from all k tests is

$$\hat{E} = E_1 + E_2 + \ldots + E_k \tag{7}$$

and , the Total Confidence of meeting the goal of reliability for the product tested is

$$\hat{c} = 1/(1 + e^{-\hat{E}}) \quad \text{[inverse of (5)]} \tag{8}$$

IN CASES OF TESTS WITHOUT FAILURES

The Evidence Accumulation is modified whenever we have a test without any failures by using only the ENTROPY TOTAL, $\mathcal{E}_{total}$, instead of the Average Entropy per Failure, $\mathcal{E}_{ave}$. However, when we do this (i.e., use $\mathcal{E}_{total}$ instead of $\mathcal{E}_{ave.}$), the formula for the corresponding Evidence for such a test without failures is

$$\text{EVIDENCE} = E = \ln\left[e^{\mathcal{E}_{total}} - 1\right] \tag{9}$$

So, whenever a certain test exhibits a certain number of life values (x_1, x_2, . . . x_N) without failures, we simply calculate

$$\mathcal{E}_{total} = \left(\frac{x_1}{\theta_{test\ goal}}\right)^{b_{test\ goal}} + \left(\frac{x_2}{\theta_{test\ goal}}\right)^{b_{test\ goal}} + \left(\frac{x_3}{\theta_{test\ goal}}\right)^{b_{test\ goal}} + \cdots + \left(\frac{x_N}{\theta_{test\ goal}}\right)^{b_{test\ goal}} \tag{10}$$

and then the Evidence from that test is given by Formula (9), and that Evidence goes into the total Equation (7).

NUMERICAL EXAMPLE OF THREE TESTS

TEST NO. 1 (Stress = 80,000 psi)	TEST NO. 2 (Stress = 90,000 psi)	TEST NO. 3 (Stress = 75,000 psi)
Goal Line Parameters $b_1 = 1.5$, $\theta_1 = 1000$ hrs.	Life Divisor = $(90000/80000)^7$ = 2.2807	Life Divisor = $(75000/80000)^7$ = .63650
	Transformed Goal : $b_2 = 1.5$, $\theta_2 = 438.46$ hrs.	Transformed Goal : $b_3 = 1.5$, $\theta_3 = 1571.09$ hrs.
TEST DATA	**TEST DATA**	**TEST DATA**
x_1 = 1050 hrs. (unfailed)	x_1 = 400 hrs. (failed)	x_1 = 1750 hrs. (unfailed)
x_2 = 975 hrs. (failed)	x_2 = 750 hrs. (failed)	x_2 = 1150 hrs. (unfailed)
x_3 = 1200 hrs. (failed)	x_3 = 300 hrs. (failed)	x_3 = 2000 hrs. (unfailed)
x_4 = 1440 hrs. (unfailed)	x_4 = 525 hrs. (unfailed)	(r_3 = 0 failures)
(r_1 = 2 failures)	x_5 = 250 hrs. (unfailed)	
	(r_2 = 3 failures)	

We want to find the composite confidence of meeting the goal for the reliability of this product , having given (from previous experimentation) , that

$$\text{LIFE} = \frac{\text{CONSTANT}}{(\text{STRESS})^7} .$$

Since the stress in Test No. 2 is 90,000 psi , we calculate

$$\theta_2 = \frac{\theta_1}{(90,000/80,000)^7} = 438.46 \text{ hrs., with the same slope } 1.5 .$$

Since the stress in Test No. 3 is 75,000 psi , we calculate

$$\theta_3 = \frac{\theta_1}{(75,000/80,000)^7} = 1571.09 \text{ hrs., with the same slope } 1.5 .$$

CALCULATING EACH TEST EVIDENCE AND THE TOTAL EVIDENCE AND THE RESULTANT ACCUMULATED CONFIDENCE OF MEETING THE PRODUCT'S GOAL

TEST NO. 1

$$(\text{Entropy Total})_1 = {}_1\mathcal{E}_{total} = \left(\frac{1050}{1000}\right)^{1.5} + \left(\frac{975}{1000}\right)^{1.5} + \left(\frac{1200}{1000}\right)^{1.5} + \left(\frac{1440}{1000}\right)^{1.5}$$

$$= 5.08120 \qquad (r_1 = 2 \text{ failures})$$

$${}_1\mathcal{E}_{ave.} = {}_1\mathcal{E}_{total}/2 = 5.08120/2 = 2.54060$$

$$\text{Evidence}_1 = E_1 = \frac{\pi}{\sqrt{3}} \sqrt{2}\,(1.54060) = 3.95178$$

TEST NO. 2

$$(\text{Entropy Total})_2 = {}_2\mathcal{E}_{total} = \left(\frac{400}{438.46}\right)^{1.5} + \left(\frac{750}{438.46}\right)^{1.5} + \left(\frac{300}{438.46}\right)^{1.5} + \left(\frac{525}{438.46}\right)^{1.5}$$

$$= \left(\frac{250}{438.46}\right)^{1.5} = 5.41523 \qquad (r_2 = 3 \text{ failures})$$

$${}_2\mathcal{E}_{ave.} = {}_2\mathcal{E}_{total}/3 = 5.41523/3 = 1.80508$$

$$\text{Evidence}_2 = E_2 = \frac{\pi}{\sqrt{3}} \sqrt{3}\,(.80508) = 2.52923$$

TEST NO. 3

$$(\text{Entropy Total})_3 = {}_3\mathcal{E}_{total} = \left(\frac{1750}{1571.09}\right)^{1.5} + \left(\frac{1150}{1571.09}\right)^{1.5} + \left(\frac{2000}{1571.09}\right)^{1.5}$$

$$= 3.23813 \qquad (r_3 = 0 \text{ failures})$$

$$\text{Evidence}_3 = E_3 = \ln\left[e^{3.23813} - 1\right] = 3.19810$$

Thus , TOTAL EVIDENCE $= \hat{E} = E_1 + E_2 + E_3 = 9.67911$,

and the resultant confidence of meeting the reliability goal is

$$\hat{c} = \frac{1}{1 + e^{-9.67911}} = .99974 \qquad \text{(ans.)}$$

CONCLUSION

We have shown a technique for accumulating evidence from a collection of tests under different conditions (with correponding goal lines) . This technique , known as the ENTROPY METHOD , is very useful and easily applied , and involves no more than two basic principles , which are

PRINCIPLE I : DAMAGE due to service is measured by ENTROPY .

PRINCIPLE II : The TOTAL EVIDENCE is the ALGEBRAIC SUM of individual evidence from separate tests.