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USING THE DUANE MODEL FOR RELIABILITY GROWTH IN PROTOTYPE VEHICLES

INTRODUCTION

The designer and manufacturer of a motor vehicle must realize an assured level of adequate reliability for a specified number of vehicle miles (say 12,000 miles) without any major failures on any main sub-assembly of his vehicle. If such an assured (desireable) level of reliability is not realized, it will turn out that too many customer complaints will result, with a consequent damage to further sales.

The main question which plagues the vehicle development process with regard to prototype vehicles is the question "How should developmental and prototype data be collected and analyzed so as to give an accurate assessment of the true state of reliability after completion of each design phase in the development process ?" It is in answer to this question that the present bulletin is intended. We shall employ the Duane Reliability Growth Model in determining the reliability level attained at any stage of development or prototype testing of vehicles .

THE DUANE RELIABILITY GROWTH MODEL

The so-called DUANE EQUATION looks as follows :

$$\lambda_x = \frac{K}{x^{\alpha}}$$

α = growth exponent

K = design constant

Where x = number of vehicle miles

λ_x = accumulated failure rate after x vehicle miles

$$= \frac{\text{No. of Failures After } x \text{ vehicle Miles}}{\text{No. of Vehicle Miles}} = \frac{r_x}{x} .$$

We define λ_c = Current Failure Rate After x Vehicle Miles

$$= \frac{dr_x}{dx} = (1 - \alpha)\lambda_x .$$

Then , it follows that

$$\text{CURRENT RELIABILITY (to } x_o \text{ vehicle miles)} = e^{-(1 - \alpha)\lambda_x x_o} \qquad \text{(I)}$$

To obtain the GROWTH EXPONENT α , simply collect data on the number of vehicle miles x versus r_x , the number of failure after x vehicle miles. Then perform Logarithmic Regression Analysis on

$$\ln\left(\frac{r_x}{x}\right) \quad \text{versus} \quad \ln x .$$

EXAMPLE

A fleet of prototype vehicles is monitored while going through a set of design phases . The data collected are listed in Table 1 below :

TABLE 1

AFTER DEDIGN PHASE	ACCUMULATED VEHICLE MILES x	ACCUMULATED FAILURES r_x
I	45,000	6
II	105,000	10
III	180,000	12
IV	270,000	12

NOTE : If the fleet consists of N vehicles , then the

$$\text{Ave. Miles per Vehicle} = \frac{\text{Accum. Vehicle Miles}}{N}$$

QUESTION : What is the Current Reliability to a target of 12,000 miles after all four design phases have been completed ?

We note that in the second column of Table 1 we list the accumulated vehicle miles. This means that

Design Phase I consisted of 45,000 vehicle miles
Design Phase II consisted of 60,000 vehicle miles
Design Phase III consisted of 75,000 vehicle miles
Design Phase IV consisted of 90,000 vehicle miles

Furthermore ,

Design Phase I showed 6 failures
Design Phase II showed 4 failures
Design Phase III showed 2 failures
Design Phase IV showed 0 failures

From Table 1 we form Table 2 by taking $\ln x$ as the transformed abscissa X and taking $\ln (r_x/x)$ as the transformed ordinate Y .

TABLE 2

AFTER DESIGN PHASE	$X = \ln x$	$Y = \ln (r_x/x)$
I	10.71422	-8.92266
II	11.56172	-9.25913
III	12.10071	-9.61581
IV	12.50618	-10.02127

We now perform LINEAR REGRESSION ANALYSIS for Y vs. X . The result is a linear relation

$$Y = -\alpha X + B$$

Where α = Growth Exponent in the Duane Model = .59613

B = Y intercept = -2.46766 Correlation Coefficient = .97827

Then , K = Design Constant in the Duane Model = e^B = .08478

Thus , for this example , The DUANE GROWTH EQUATION is

$$\lambda_x = \frac{.08478}{x^{.59613}}$$

For a target of 12,000 vehicle miles , the Current Reliability (after Design Phase IV) is (according to Equation (I) on page 2)

$$R_{IV}(12,000) = e^{-(1 - .59613)\left(\frac{.08478}{270,000^{.59613}}\right)(12,000)}$$

$$= .78849 \quad \text{(Reliability after Phase IV)}$$

Furthermore ,

$$R_{III}(12,000) = e^{-(1 - .59613)\left(\frac{.08478}{180,000^{.59613}}\right)(12,000)}$$

$$= .73888 \quad \text{(Reliability after Phase III)}$$

and ,

$$R_{II}(12,000) = e^{-(1 - .59613)\left(\frac{.08478}{105,000^{.59613}}\right)(12,000)}$$

$$= .65883 \quad \text{(Reliability after Phase II)}$$

and

$$R_{(I)}(12,000) = e^{-(1 - .59613)\left(\frac{.08478}{45,000^{.59613}}\right)(12,000)}$$

$$= .50082 \quad \text{(Reliability after Phase I)}$$

CONCLUSION

The example , according to the Duane Model , shows a reliability growth from only 50% (to 12,000 miles) in Phase I to a reliability of 78.8% (to 12,000 miles) in Phase I V . For still higher reliability there must be still more improvement in later phases if we want to reach 90% or more in reliability .

The Duane Reliability Growth Model simplifies the entire decision process as the development stages expire . The designer is free to make design changes as frequently as desired as long as he is able to collect sufficient data within each design phase .