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PRINCIPLES OF QUANTITATIVE RELIABILITY MANAGEMENT

INTRODUCTION

In designing a reliability testing and development program it has been repeatedly emphasized at Detroit Research Institute that the FIRST RULE to be observed is the following :

RULE # 1 : In order to design an effective program of reliability testing we must know the financial parameters of our situation . These are

(A) The DOLLAR GAIN PER GOOD ITEM , i. e., the net income from each item which complies with the reliability goal .

(B) The DOLLAR LOSS PER BAD ITEM , i. e., the dollar losses suffered for each item which fails to comply with the reliability goal (such as a warranty promise) .

QUANTITATIVE RELIABILITY MANAGEMENT is based on the foundation of these FINANCIAL PARAMETERS , as well as SPECIFIED PROFITABILITY FACTOR , which represents the RATIO BETWEEN EXPECTED DOLLAR GAINS AND EXPECTED DOLLAR LOSSES. A profit making organization wants this profitability factor (call it K) to be greater than unity. If $K = 1$, then the business is just breaking even, whereas if $K < 1$, then there is a net loss due to excessive unreliability .

THE QUANTITATIVE RELIABILITY MANAGEMENT EQUATIONS

There are two types of mathematical equations pertaining to Quantitative reliability management .

These are :

(A) Sample Size Equations

(B) Confidence Equations for different profitability factors .

Sample size equations are functions of

(1) The number of Defectives , D , observed in a test to a reliability goal .

(2) The dollar gain , G , per good item .

(3) The dollar loss , L , per bad item .

(4) The desired median profitability factor , $K_{.50}$.

Knowing D, G, L, and $K_{.50}$ we can write the equation for the required sample size as

$$N = \frac{(D + .7) K_{.50} L}{G} + (D - .7) \qquad \text{(I)}$$

This equation (I) is derived using Benard's Formula for the Median Rank of a target to which there are D defectives observed in N trials .

This Median Rank is $F_{.50} = \frac{D + 1 - .3}{N + 1 + .4} = \frac{D + .7}{N + 1.4}$.

Hence , the Median Reliability is $R_{.50} = 1 - \frac{D + .7}{N + 1.4}$.

For T items produced the total dollars gained is $T R_{.50} G$, while the total dollars lost due to unreliability is $T F_{.50} L$.

If we desired a Median Profitability Factor $K_{.50}$, then it must follow that

$$K_{.50} = \frac{T R_{.50} G}{T F_{.50} L} = \frac{R_{.50} G}{F_{.50} L} \tag{II}$$

Equation (II) leads to equation (I) when we put $F_{.50} = \frac{D + .7}{N + 1.4}$

and $R_{.50} = 1 - \frac{D + .7}{N + 1.4}$ thus ,

$$K_{.50} = \frac{\left(1 - \frac{D + .7}{N + 1.4}\right) G}{\left(\frac{D + .7}{N + 1.4}\right) L} = \frac{(N - D + .7) G}{(D + .7) L}$$

Solving this for the sample size N yields $N = \frac{(D + .7) K_{.50} L}{G} + (D - .7)$,

which is equation (I) .

CONFIDENCE EQUATIONS FOR PROFITABILITY FACTORS

We know that the confidence level for $K_{.50}$ is .50 by definition . This is the same confidence level for the reliability $R_{.50} = 1 - D + .7/N + 1.4$. Now if we desire the confidence level c for any other profitability factor of magnitude $K_c \neq K_{.50}$ we must first determine the reliability at that value of K_c. This reliability R_c is such that for total production T we have

$$T R_c G = K_c T F_c L = K_c T (1 - R_c) L$$

or $$R_c G = K_c (1 - R_c) L = K_c L - K_c R_c L$$

or $$R_c (K_c L + G) = K_c L$$

or $$R_c = K_c L / K_c L + G \quad .$$

We know that the c-Rank Theorem states that

$$R_c = 1 - \text{c-Rank of } (D + 1)^{th} \text{ order statistic in } (N + 1) \tag{III}$$

Hence , the confidence level c for the profitability factor K_c must be such that it satisfies this c-Rank Theorem (equation III above) .

AN EXAMPLE OF THE APPLICATION OF THE QUANTITATIVE RELIABILITY MANAGEMENT EQUATIONS

Suppose a manufacturer has determined that

(a) Dollar Gain per Good Item = G = \$80 .

(b) Dollar Loss per Bad Item = L = \$500 .

(c) Median Profitability Factor Desired = $K_{.50} = 3$.

(d) Reliability Goal = 5000 Hours .

If D = 2 defectives are permitted in a test to the reliability goal of 5000 hours, how large should the test sample be ? How much confidence would there be for (1) K = 1 (breaking even) ?

(2) K = 2 (twice as much gained as lost) ?

SOLUTION

$$N = \frac{(D + .7)\, K_{.50}\, L}{G} + (D - .7)$$

$$N = \frac{(2.7)\,(3)\,(500)}{80} + 1.3 = 51.925 \text{ or } N = 52 \text{ (next integer)}.$$

Thus , 52 items must be tested to the goal of 5000 hours , with D = 2 deffectives permitted . The confidence c_1 of breaking even will such that , for $K_{c_1} = 1$,

c_1-Rank of 3rd in 53 = 1 - L/L + G = 1 - 500/580 = .137931

Thus , $c_1 = .9757$ (Answer to (1)) .

On the other hand , the confidence c_2 of having a profitability factor $K_{c_2} = 2$ is such that

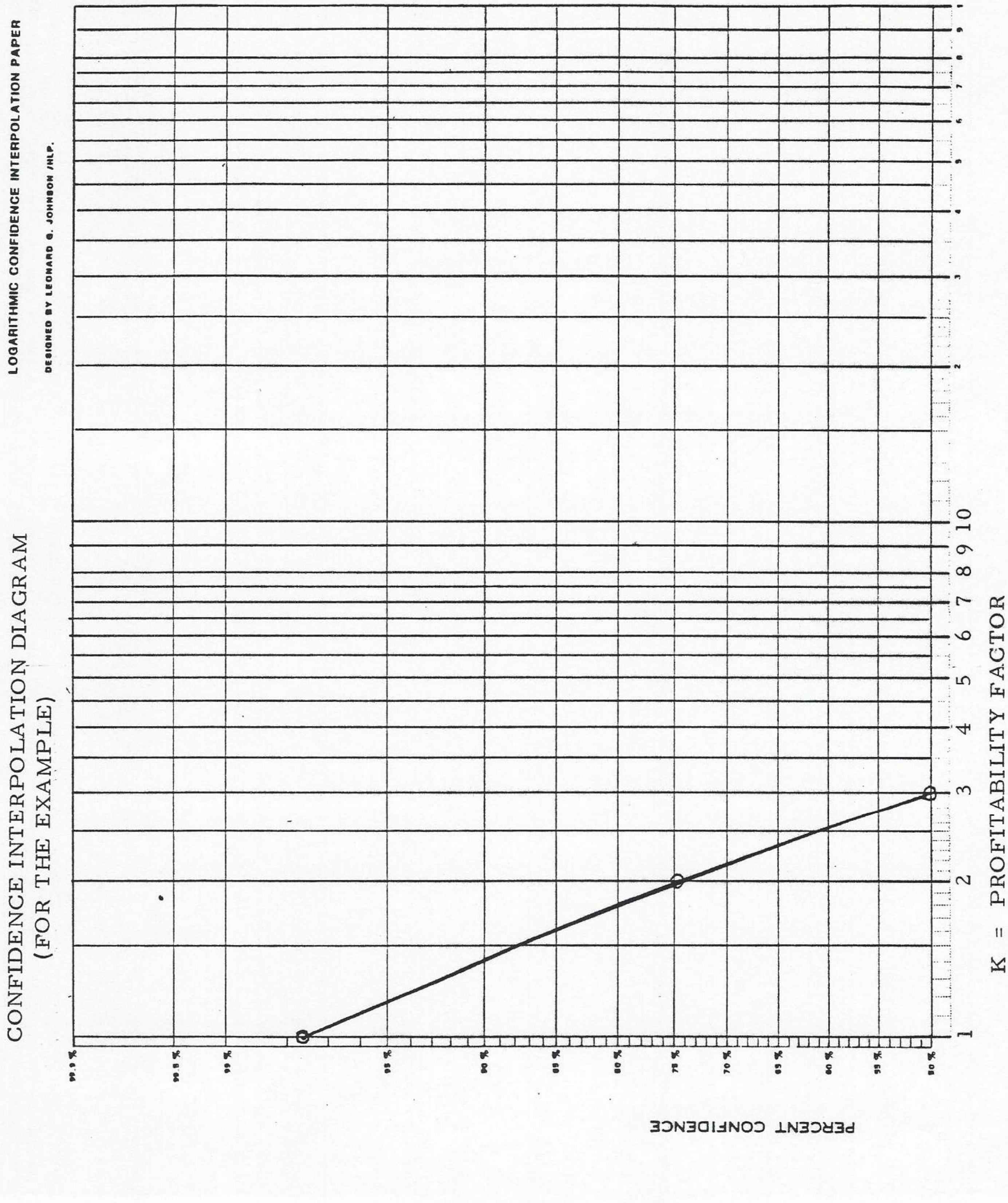
c_2-rank of 3rd in 53 = 1 - 2L/2L + G = 1 - 1000/1080 = .074074

This yields $c_2 = .7476$ (Answer to (2)) .

BY DEFINITION : The confidence for $K_{.50} = 3$ is $c_3 = .50$.

CONCLUSION : This example shows the general procedure in testing programs based on the principles of quantitative reliability management . The CONFIDENCE INTERPOLATION DIAGRAM of Figure 1 graphically summarizes the situation for this particular example .

FIGURE 1


CONFIDENCE INTERPOLATION DIAGRAM
(FOR THE EXAMPLE)
LOGARITHMIC CONFIDENCE INTERPOLATION PAPER
DESIGNED BY LEONARD G. JOHNSON /HLP.
K = PROFITABILITY FACTOR
1
2
3
4
5
6
7
8
9
10
PERCENT CONFIDENCE