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COSTING OUT THE RISK IN PRODUCT RELIABILITY CERTIFICATION

INTRODUCTION

The first rule of designing an adequate reliability testing program in the development of a new design in the automotive industry (or any other industry) is the rule which states that a test sample size must be large enough to yield sufficient confidence of making a reliability promise realizable with positive profit over and above the losses incurred by failure to keep the reliablity promise.

In this bulletin we shall illustrate the proper method of calculating the required confidence index for a reliability testing program, and, consequently, the sample size dictated by the safety factor needed to avoid losses due to failures beyond the promised reliability level of a consumer product.

A TYPICAL PROMISE IN AN ILLUSTRATED SALES SITUATION

Suppose it is promised that the B_{10} life of a certain automotive component is at least 50,000 miles. In reliability terminology, this can be stated as follows: $R_c(50{,}000 \text{ Miles}) \geq .90$,

which states that the reliability to 50,000 miles is at least 90% with confidence c. This means that the risk of the B_{10} life falling below 50,000 miles is only $(1 - c)$. So, for example, if $R_{.95}(50{,}000 \text{ Miles}) \geq .90$, it means that the risk of the B_{10} life falling below 50,000 miles is only 5%. In other words, if 20 dealers were each sold 10,000 such components, only one dealer out of the twenty would have over 10% of his components failing prior to 50,000 miles. The median number of such components not lasting 50,000 miles would then be

$$10{,}000\ F_{.975}(50{,}000 \text{ Miles}) = 10{,}000\left[1 - R_{.975}(50{,}000 \text{ Miles})\right],$$

where

$R_{.975}(50{,}000 \text{ Miles})$ = Reliability to 50,000 Miles with 97.5% confidence

Suppose the manufacturer's test sample size is N = 30. Then, by Log-Parametric Theory,

$$R_{.95}(50,000\ \text{Mi.}) = \left[R_{.50}(50,000\ \text{Mi})\right]^{\left(\frac{.95}{1-.95}\right)^{\frac{.55}{\sqrt{30}}}} = \left[R_{.50}(50,000\ \text{Mi})\right]^{19^{\frac{.55}{\sqrt{30}}}}$$

$$R_{.95}(50,000\ \text{Mi.}) = .9 \quad \text{and}$$

$$R_{.975}(50,000\ \text{Mi.}) = \left[R_{.50}(50,000\ \text{Mi})\right]^{\left(\frac{.975}{1-.975}\right)^{\frac{.55}{\sqrt{30}}}}$$

$$R_{.975}(50,000\ \text{Mi.}) = \left[R_{.50}(50,000\ \text{Mi})\right]^{39^{\frac{.55}{\sqrt{30}}}}$$

So,

$$R_{.975}(50,000\ \text{Mi.}) = .9^{(39/19)^{.55/\sqrt{30}}}$$

NOTE: (.975 is the median distance from .95 to 1.00)

$$\therefore \quad R_{.975}(50,000\ \text{Mi.}) = .892927252$$

so, $F_{.975}(50,000\ \text{Mi.}) = .107072748$

and $10,000\ F_{.975}(50,000\ \text{Mi.}) = 1071$

Thus, one dealer would sell 71 extra components not surviving 50,000 miles (over and above the 1000 permitted, i.e., over the 10% which are not promised to survive 50,000 miles.).

Now, suppose that

(a) The Component Mfg. Cost = \$30.00

(b) The Component Selling Price = \$30.50

This means that on 200,000 components which are sold in totality to 20 dealers (10,000 to each), the manufacturer's profit would be

$$200,000\ (30.50 - 30.00) = \$100,000$$

If the manufacturer loses \$200 on each of the 71 extra that failed to last 50,000 miles, this would be a total loss of 200 x 71 = \$14,200, including the manufacturing cost of the replacement component, together with labor costs, shipping costs, and storage costs, as well as customer dissatisfaction costs.

Thus in this case, we conclude that the manufacturer has a monetary safety factor given by

$$K = \frac{\$100,000}{\$14,200} = 7.04 .$$

THE EFFECT OF REDUCING THE CONFIDENCE TO C = .70

In the example just discussed, if the manufacturer found that a sample of 30 test components yielded only 70% confidence (instead of 95% confidence) that the B_{10} life would be at least 50,000 miles, then the monetary safety factor would be affected as follows :

The relevant factors to be calculated are now

(a) $$R_{.70}(50,000 \text{ Mi.}) = \left[R_{.50}(50,000 \text{ Mi.})\right]^{(7/3)^{.55/\sqrt{30}}} = .9$$

(b) $$R_{.85}(50,000 \text{ Mi.}) = \left[R_{.50}(50,000 \text{ Mi.})\right]^{(85/15)^{.55/\sqrt{30}}}$$

so, $$R_{.85}(50,000 \text{ Mi.}) = .9^{(85/15 \times 3/7)^{.55/\sqrt{30}}} = .9^{(17/7)^{.55/\sqrt{30}}}$$

$$R_{.85}(50,000 \text{ Mi.}) = .891206626 .$$

NOTE : (.85 is the median distance from .70 to 1.00)

So, the number of extra components not lasting 50,000 miles (for those dealers not realizing the promise that B_{10} life $\geq$ 50,000 miles) is per such dealer

$$10,000\ F_{.85}(50,000 \text{ Mi.}) = 10,000\,(1 - .891206626)$$
$$= 10,000\,(.108793374) = 1088,$$

which is 88 extra over the 1000 permitted.

Since the promise of B_{10} life of 50, 000 miles was made with 70% confidence, this means that 30% of the dealers would have a median of 88 components not living up to the B_{10} life guarantee , this is , 30% of 20 , or 6 dealers would demand reimbursement for a total of

$$6 \times 88 = 528 \text{ replacements} .$$

If each replacement costs the manufacturer \$200 (as assumed before), this would be a loss to the manufacturer of

$$\$200 \times 528 = \$105,600$$

Thus, the manufacturer's original profit of \$100, 000 would be wiped out by using a confidence index of only 70% in the reliability promise , which said ,

$$R_{.70}(50,000 \text{ Mi.}) \geq .90$$

or that the B_{10} life $\geq$ 50, 000 miles with 70% confidence. So, there is no profitability in testing only 30 if they yield only 70% confidence.

THE CONFIDENCE LEVEL NEEDED IN ORDER TO MAKE K = 2.0

Let us take the same example of promising a B_{10} life of at least 50, 000 miles based on a test sample size of N = 30. The question we now raise is the following :

If a monetary safety factor of K = 2 is desired , what level of confidence c must be demonstrated for the reliability promise which states that $R_c(50,000 \text{ Mi.}) \geq .9$?

SOLUTION

The formulas for the relevant reliabilities are

$$R_c(50,000 \text{ Mi.}) = \left[R_{.50}(50,000 \text{ Mi.})\right]^{(c/1-c)^{.55/\sqrt{30}}} = .9$$

$$R_{(1+c)/2}(50,000 \text{ Mi.}) = \left[R_{.50}(50,000 \text{ Mi.})\right]^{(1+c/1-c)^{.55/\sqrt{30}}} = .9^{(1+c/c)^{.55/\sqrt{30}}}$$

NOTE: ((1 + c)/2 is at half the distance from c to 1.00)

So, the median number of components needing replacement by the manufacturer is

$$\xi_c = \underbrace{20}_{\text{Number of Dealers}}(1 - c)\left\{\underbrace{10,000}_{\text{No. Sold to Each Dealer}}\left[1 - .9^{\left(\frac{1+c}{c}\right)^{\frac{.55}{\sqrt{30}}}}\right] - \underbrace{1000}_{\text{10\% of the 10,000 Sold to Each Dealer}}\right\} \qquad \text{(A)}$$

Using formula (A) for ξ_c for various values of the confidence $\underline{c}$, we form the following table (assuming \$200 = cost of each replacement):

c (Confidence)	$\$200\,\xi_c$ (Total Replacement Cost)	$K = \left(\frac{\$100,000}{\$200\,\xi_c}\right)$ (Monetary Safety Factor)
.80	\$64,071	1.56
.85	\$46,015	2.17
.84	\$49,600	2.016

Thus, if 30 test specimens yield a confidence index of 84%, the manufacturer will realize a monetary safety factor K which is approximately K = 2.

Furthermore, since 70% confidence caused the replacement cost to become \$105,600 (i.e., just slightly more than the \$100,000 original profit), we estimate that

For c = .71, the K factor would be just about equal to 1, i.e., to break even, the manufacturer could get along with a confidence index of 71%.

THE REQUIRED B_{10} LIFE ON A MEDIAN RANK WEIBULL PLOT

QUESTION: For a Weibull slope of 1.5, and a Monetary Safety Factor of K = 2, and a sample size of 30 failures in a test of the same component, how much B_{10} life must the median rank Weibull plot show ?

SOLUTION

We just discovered that the required confidence level is c = .84 (approx.) in order to yield a Monetary Safety Factor of K = 2 from a sample of 30 test specimens.

Hence, the μ factor is

$$\left(\frac{c}{1-c}\right)^{.55/\sqrt{30}} = \left(\frac{.84}{.16}\right)^{.55/\sqrt{30}} = (5.25)^{.1004156802} = 1.181182372 .$$

Now, according to Log-Parametric Rank Theory, the Life Ratio

$$\rho = \frac{{}_{.50}B_{10}}{{}_{.84}B_{10}} \quad \text{must be}$$

equal to $\mu^{1/b} = (1.181182372)^{1/1.5} = 1.117406786$.

$\therefore$ ${}_{.50}B_{10}$ (on the median rank plot) = 50,000 x 1.117406786 = 55,870 Miles (answer)

CONCLUSION

By specifying the Monetary Safety Factor (K) desired on any reliability promise, it is possible to determine the appropriate values of sample size and confidence required to cover replacement costs due to extra failures (beyond the promised maximum number), provided we know the profit per piece originally sold, as well as the cost per replacement required. Furthermore, the amount of shift to the right required beyond target in the Weibull plot can also be determined for any combination of sample size and confidence .

APPENDIX

How a B_{10}- Life promise of 50, 000 miles with a given confidence <u>c</u> affects a warranty promise of 12, 000 miles .

In the example just discussed it was concluded that with a Weibull slope of 1. 5 the median Weibull line crossed the 10th percentile at 55, 870 miles when the confidence was c = . 84.

In that same median Weibull plot, the fraction failing by 12, 000 miles would be

$$F_{.50}(12,000 \text{ Mi.}) = 1 - e^{-(12,000/\theta)^{1.5}}$$

where

$$\theta = \frac{55,870}{\ln\left(\frac{1}{.9}\right)^{1/1.5}} = 250,453 \text{ miles .}$$

Thus

$$F_{.50}(12,000 \text{ Mi.}) = 1 - e^{-(12,500/250,453)^{1.5}} = .01043$$

Thus, when c = . 84 for a B_{10} life promise of 50, 000 miles, it turns out that the median fraction of the items not lasting 12, 000 miles is 1. 043% . For a total of 200, 000 items sold , this amounts to a total of . 0143 (200, 000) = 2086 warranty claims. At $200 expense per warranty claim, this is an expense of $200 x 2086 = $417, 200 .

So, for a Monetary Safety Factor of K = 2, the original profit per sold item must be such that for 200, 000 sold, the total original profit is 2 x $417, 200 = $834, 400 , i. e., a profit of 834, 400/200, 000 = $4. 18 per item. Thus, as far as a 12, 000 mile warranty is concerned, the selling price should be set at $34. 18 per item (if the manufacturing cost per item is $30. 00) .