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THE SECRET OF THE GROWTH OF EVIDENCE IN THE TESTING OF SCIENTIFIC HYPOTHESES

INTRODUCTION

A basic question in scientific investigations is the question of gathering evidence in favor of a theory or hypothesis. What is the secret of demonstrating evidence in such an investigation? It is desirable to be able to quantify the growth of evidence and to develop a straightforward technique of monitoring such a growth of evidence. Once such a technique is developed it can be used to design proper experimental tests of scientific hypotheses, and it can even tell us how large test samples should be before testing can be concluded with adequate confidence about the validity of the hypothesis being investigated.

The purpose of this bulletin is to clarify the proper procedure to be used in measuring the growth of evidence, together with a review of the basic definition of what is called the mathematical concept of evidence.

ENTROPY ---- THE ENEMY

Whenever we have a probability in favor of some event, we call the LOGARITHM OF THAT PROBABILITY'S RECIPROCAL the ENTROPY AGAINST that event. For example, if the survival probability of a product after x hours of service is denoted by R(x) (i.e., R(x) = Reliability at Time x) , then the ENTROPY AGAINST SURVIVAL at time x is

$$\mathcal{E}(x) = \ln \frac{1}{R(x)} .$$

Likewise, if F(x) is the PROBABILITY OF THE SAME PRODUCT FAILING within x hours of service, we define the ENTROPY AGAINST FAILURE in time x by the formula

$$\overline{\mathcal{E}}(x) = \ln \frac{1}{F(x)} .$$

Since $F(x) = 1 - R(x)$, we could also write

$$\overline{\mathcal{E}}(x) = \ln \frac{1}{1 - R(x)} .$$

Thus, ENTROPY is always an ENEMY which is against the event whose probability we are considering.

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EVIDENCE ----- A BATTLE BETWEEN ENTROPIES

Any time that we are dealing with the occurrence of a certain event and its probability we must at the same time deal with the NON-OCCURRENCE PROBABILITY of the same event.

The OCCURRENCE PROBABILITY has its enemy (preventer) called ENTROPY AGAINST OCCURRENCE, while the NON-OCCURRENCE PROBABILITY has its enemy (preventer) called the ENTROPY AGAINST NON-OCCURRENCE.

EVIDENCE in favor of the occurrence of an event whose OCCURRENCE PROBABILITY is P and whose NON-OCCURRENCE PROBABILITY is $(1 - P)$, is defined as follows:

EVIDENCE IN FAVOR OF THE EVENT

= Entropy Against Non-Occurrence - Entropy Against Occurrence

$$= \ln \frac{1}{1 - P} - \ln \frac{1}{P}$$

$$= - \ln(1 - P) + \ln P$$

$$= \ln\left(\frac{P}{1 - P}\right) .$$

Thus, from this ENTROPY DIFFERENCE , we conclude that

EVIDENCE in favor of the event = LOGARITHM of ODDS in Favor of Event

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AN IMPORTANT PROPERTY ----- THE ADDITIVITY OF INDEPENDENT EVIDENCES

The accumulation of evidence in favor of any hypothesis is accomplished by simple addition in an algebraic sense of two or more bits of evidence.

For example, if one collected data set yields a confidence index C_1 in favor of some hypothesis (such as the claim that DESIGN II has longer average life than DESIGN I) and a second collected data set yields a confidence index C_2 in favor of the same hypothesis, then we have

EVIDENCE from DATA SET I :
$$E_1 = \ln\left(\frac{C_1}{1 - C_1}\right)$$

EVIDENCE from DATA SET II:
$$E_2 = \ln\left(\frac{C_2}{1 - C_2}\right)$$

By addition : TOTAL EVIDENCE
$$= E_1 + E_2 = \ln\left(\frac{C_1}{1 - C_1}\right) + \ln\left(\frac{C_2}{1 - C_2}\right)$$

i.e.,
$$E_{Total} = \ln\left[\frac{C_1 C_2}{(1 - C_1)(1 - C_2)}\right]$$

Such an additive process can be continued for as many data sets as we desire, thus accumulating evidence as we go along. This is the whole secret of the growth of evidence by SEQUENTIAL ANALYSIS. The TOTAL CONFIDENCE then is

$$C_{Total} = \frac{1}{1 + EXP\,(- E_{Total})}$$

A NUMERICAL EXAMPLE OF ACCUMULATION OF EVIDENCE

PROBLEM: A test engineer runs a life testing program in which he is accumulating evidence that the B_{10} Life of his product is at least 1000 hours, as required by the customer.

In his first test set-up he tested 20 of the items and the Weibull plot showed a slope of 1.5 and a MEDIAN LIFE of 4000 hours.

This yields EVIDENCE (in the parametric sense)

$$E_1 = \frac{b \sqrt{N} \ln \rho}{.55}$$

where $b = 1.5$; $N = 20$; $\rho = \left(\frac{4000}{1000}\right)\left(\frac{\ln \frac{1}{.9}}{\ln 2}\right)^{\frac{1}{1.5}} = 1.13927$

Thus, $E_1 = \frac{1.5 \sqrt{20} \ln(1.13927)}{.55} = 1.59035$

(This amounts to a confidence of $C_1 = \frac{1}{1 + EXP(-1.59035)} = .83067$)

In his second test set-up the engineer tested 10 items which plotted on Weibull paper with a slope of 1.5 and a MEDIAN LIFE of 3790 hours.

This implies an EVIDENCE $E_2 = \frac{b \sqrt{N} \ln \rho}{.55}$

where $b = 1.5$; $N = 10$; $\rho = \left(\frac{3790}{1000}\right)\left(\frac{\ln \frac{1}{.9}}{\ln 2}\right)^{\frac{1}{1.5}} = 1.07946$

Thus, $E_2 = \frac{1.5 \sqrt{10} \ln(1.07946)}{.55} = .65943$

(This amounts to a confidence of $C_2 = \frac{1}{1 + EXP(-.65943)} = .65913$)

Thus, the TOTAL EVIDENCE that the B_{10} Life is at least 1000 hours is

$$E_{Total} = E_1 + E_2 = 1.59035 + .65943 = 2.24978.$$

This yields a TOTAL CONFIDENCE of

$$C_{Total} = \frac{1}{1 + EXP(-2.24978)} = .90463 \quad \text{(ans.)}$$

Thus, the secret is to ADD TOGETHER ALL EVIDENCES (E_1 , E_2, . . . , E_k) into a Total

$$E_{Total} = E_1 + E_2 + \ldots + E_k \ .$$

Then, the TOTAL CONFIDENCE in favor of the hypothesis is

$$C_{Total} = \frac{1}{1 + EXP(-E_{Total})}$$

<u>NOTE:</u> This last formula is simply the INVERSE of the relation $E_{Total} = \ln\left(\frac{C_{Total}}{1 - C_{Total}}\right)$