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PROPERTIES OF TWO - COMPONENT SERIES CURVES IN WEIBULL ANALYSIS

1. A "Series" curve is generated on probability paper (such as Weibull paper) whenever the individual data points represent random occurrences of two or more modes of failure, or two or more failing components of an assembly consisting of such components in series.

The "Series" curve, therefore, represents ASSEMBLY failures for an assembly consisting of components in SERIES.

2. For example, look at FIGURE 1 below.

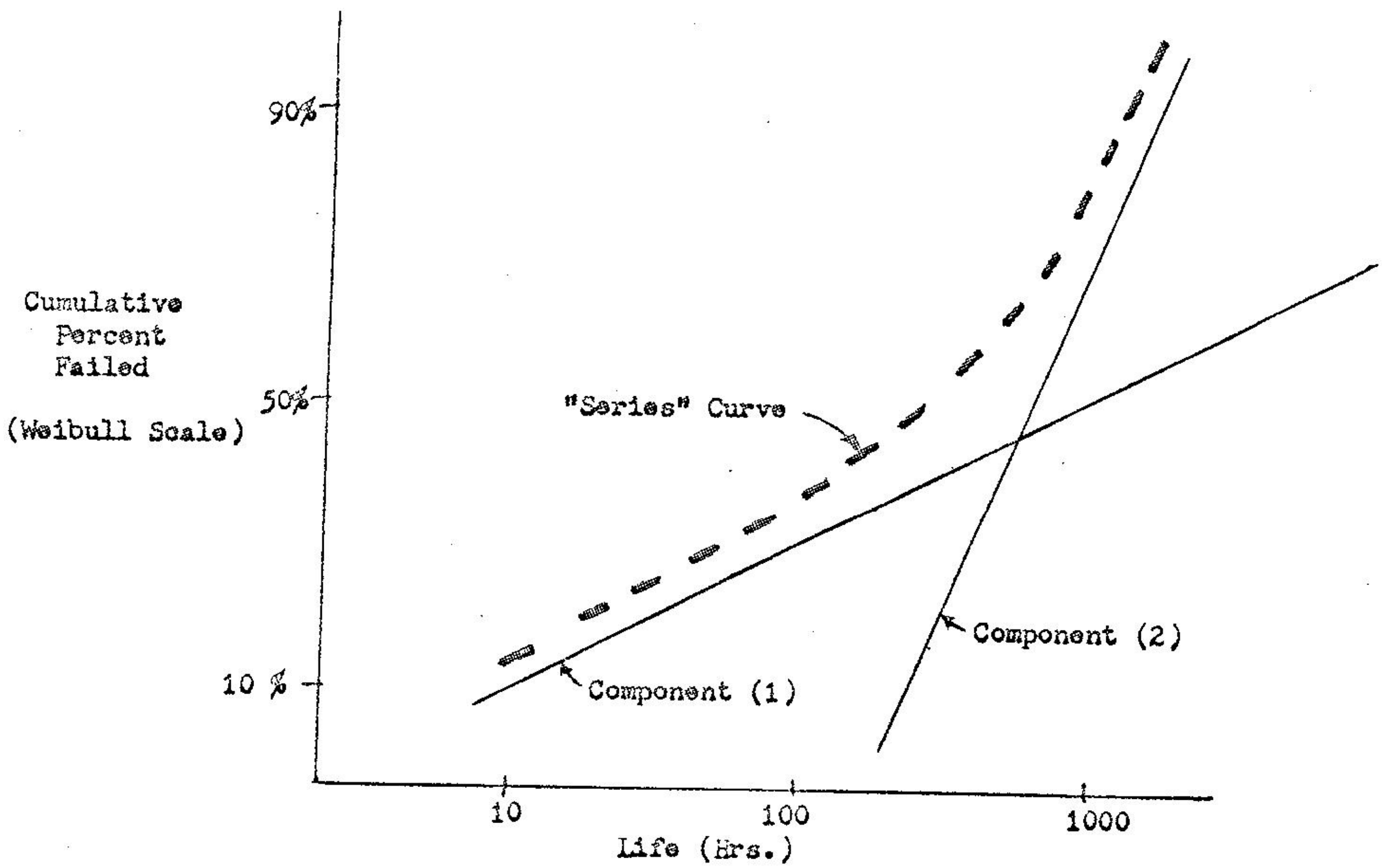

FIGURE 1

In this illustration, component (1) and component (2) are in series, and their Weibull lines are straight. The assembly Weibull plot, however, is not straight, but is represented by the dotted "Series" curve.

(3.) When the Weibull lines for (1) and (2) are straight, but intersecting, the "Series" curve cannot be STRAIGHT .

(4.) When the Weibull lines for (1) and (2) are straight, but parallel, (FIGURE 2), the "Series" curve is also STRAIGHT, and of the same WEIBULL SLOPE.

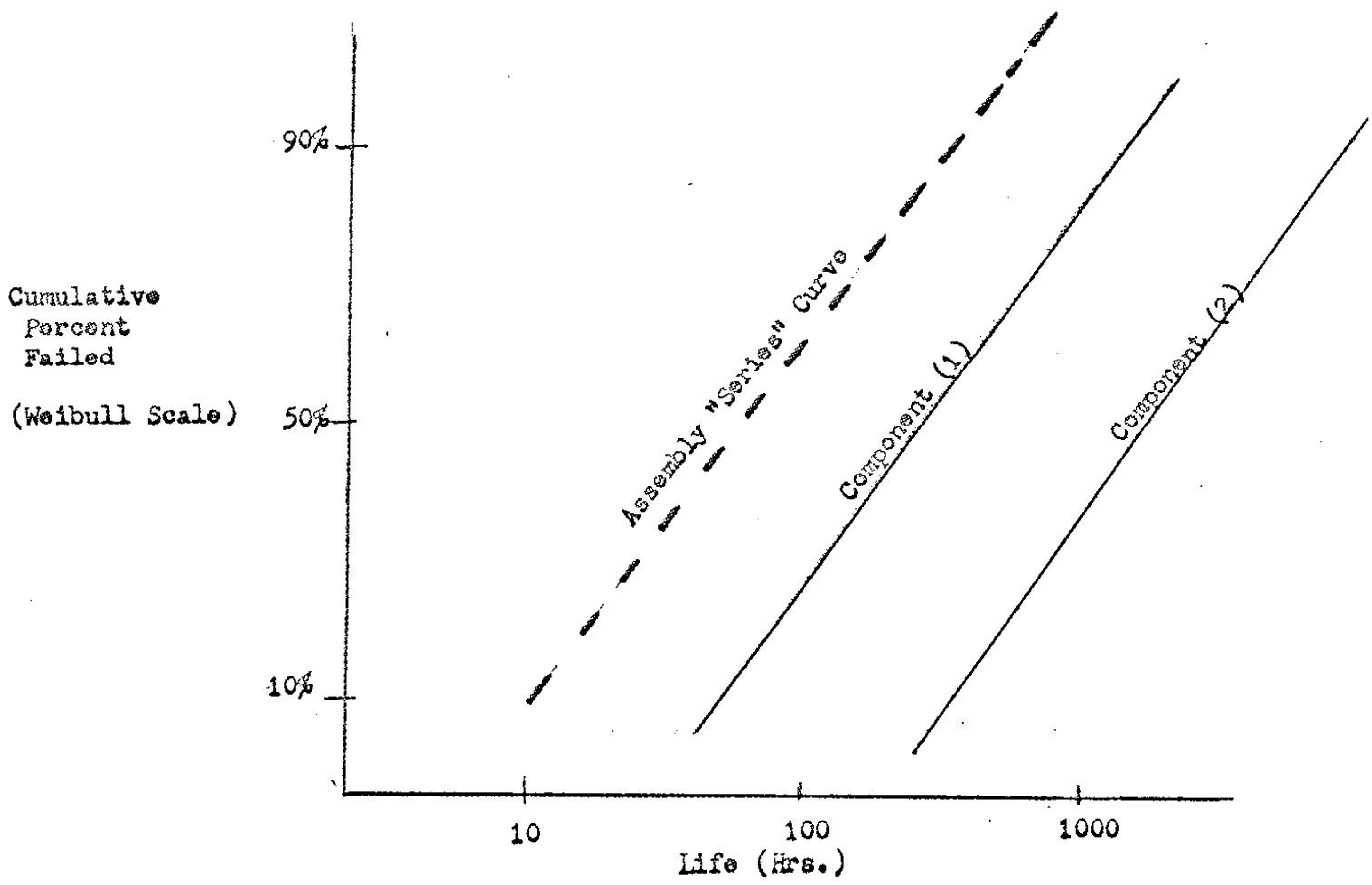

FIGURE 2

5. A "Series" curve is always to the LEFT of all component curves.

Let $F_1(x)$ = CDF of (1)

Let $F_2(x)$ = CDF of (2)

Then the CDF of the assembly "Series" curve is

$$\hat{F}(x) = 1 - \left[1 - F_1(x)\right]\left[1 - F_2(x)\right].$$

What rule is used?

The PRODUCT rule for the RELIABILITY of components in SERIES.

6. Suppose $F_1(x) = 1 - \text{EXP}(-(x/\theta_1)^{b_1})$

and $F_2(x) = 1 - \text{EXP}(-(x/\theta_2)^{b_2})$ $(b_2 > b_1)$

If these two Weibull components (or modes of failure) are in series, then the "Series" curve formula is

$$\hat{F}(x) = 1 - \text{EXP}\left[-\left(\frac{x}{\theta_1}\right)^{b_1} - \left(\frac{x}{\theta_2}\right)^{b_2}\right].$$

This "Series" curve has the following special properties:

PROPERTY I : The two component lines intersect at abscissa

$$x_0 = \left(\frac{\theta_2^{\,b_2}}{\theta_1^{\,b_1}}\right)^{\frac{1}{b_2 - b_1}}$$

(7.) PROPERTY II : The point of maximum curvature on the "Series" curve has abscissa

$$x_1 = \left(\frac{\theta_2^{\,b_2}}{\lambda \, \theta_1^{\,b_1}} \right)^{\frac{1}{b_2 - b_1}}$$

where λ is the positive root of the cubic

$$(1+b_1^2)\lambda^3 + (1 + 2b_1^2 - b_1 b_2)\lambda^2 + (b_1 b_2 - 2 b_2^2 - 1)\lambda - (1 + b_2^2) = 0 .$$

A good empirical formula for λ is

$$\lambda = 1 + (b_2 - b_1) \sqrt[3]{\frac{\text{LOG}\left(\frac{b_2}{b_1}\right)}{\text{LOG } 3}}$$

PROPERTY III : The minimum radius of curvature (ρ) at the point of maximum curvature on the assembly curve is given by the formula

$$\rho = \frac{\left[(1+b_1^2)\left(\frac{x_1}{\theta_1}\right)^{2b_1} + 2(1+b_1 b_2)\left(\frac{x_1}{\theta_1}\right)^{b_1}\left(\frac{x_1}{\theta_2}\right)^{b_2} + (1+b_2^2)\left(\frac{x_1}{\theta_2}\right)^{2b_2}\right]^{\frac{3}{2}}}{(b_2 - b_1)^2\left[\left(\frac{x_1}{\theta_1}\right)^{2b_1}\left(\frac{x_1}{\theta_2}\right)^{b_2} + \left(\frac{x_1}{\theta_1}\right)^{b_1}\left(\frac{x_1}{\theta_2}\right)^{2b_2}\right]}$$

where x_1 = Abscissa of the point of maximum curvature (as given above) .

8. PROPERTY IV : The slope of the assembly curve at the point of maximum curvature is given by the formula

$$\text{SLOPE} = \left(\frac{\lambda}{1+\lambda}\right) b_1 + \left(\frac{1}{1+\lambda}\right) b_2$$

where λ is defined as in 7.

9. PROPERTY V : The slope of the segment joining (x_0, y_0) and (x_1, y_1), i.e., the segment joining the intersection point and the point of maximum curvature, is given by the formula

$$\text{SLOPE} = b_2 + (b_1 - b_2) \frac{\text{LOG}(1 - \lambda)}{\text{LOG}\,\lambda}$$

where λ is defined as in 7.

(10.) Some other special relationships are

$$\left(\frac{X_0}{\theta_1}\right)^{b_1} = \left(\frac{X_0}{\theta_2}\right)^{b_2} = \left(\frac{\theta_2}{\theta_1}\right)^{\frac{b_1 b_2}{b_2 - b_1}}$$

$$\left(\frac{X_1}{\theta_1}\right)^{b_1} = \frac{1}{\lambda^{\frac{b_1}{b_2 - b_1}}} \left(\frac{\theta_2}{\theta_1}\right)^{\frac{b_1 b_2}{b_2 - b_1}}$$

$$\left(\frac{X_1}{\theta_2}\right)^{b_2} = \frac{1}{\lambda^{\frac{b_2}{b_2 - b_1}}} \left(\frac{\theta_2}{\theta_1}\right)^{\frac{b_1 b_2}{b_2 - b_1}}$$

Minimum radius of curvature $= \rho = \dfrac{\left[(\lambda + 1)^2 + (b_1\lambda + b_2)^2\right]^{3/2}}{\lambda(\lambda + 1)(b_2 - b_1)^2}$

NOTE: x_o = abscissa of the intersection point

x_1 = abscissa of the point of maximum curvature

λ was defined in (7.) .

$(b_2 > b_1)$