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APPLYING THE AVERAGE ENTROPY PER FAILURE TECHNIQUE TO LIFE DATA WITH TIME GAPS

INTRODUCTION

A most useful technique for judging as to whether or not a given life data set indicates compliance to a given reliability goal line is the so-called AVERAGE ENTROPY PER FAILURE TECHNIQUE. The use of the technique has been outlined for machines whose histories are given, starting at time zero. *

In this bulletin we consider the application of the technique to data on machines whose histories are recorded only some time later than time zero. For example, we might be concerned with some older machine models, on which data collecting was not initiated until the machines under consideration had already been in use for 4000 hours. How is such a data set to be analyzed for compliance to a goal? We answer this question in the subsequent pages.

ENTROPY ACCUMULATION ON A GOAL LINE AS MEASURED FROM A FIXED STARTING TIME

Let us suppose that in Figure 1 below the line AB represents a Reliability Goal for a certain type of machine or system. The abscissa is the Time in Service, While the ordinate represents the Fraction Failed.

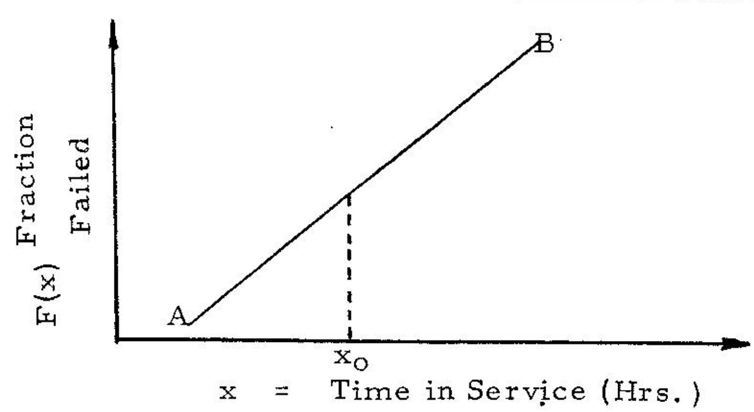

Figure 1

* See DRI's Statistical Bulletin - Volume 7 ; Bulletin 8 ; February, 1978

Suppose , furthermore , that we have some service history of five machines from time x_o and onward , but no history prior to time x_o .

Now ,

Let x_1 = Hrs. of total service on Machine #1

Let x_2 = Hrs. of total service on Machine #2

Let x_3 = Hrs. of total service on Machine #3

Let x_4 = Hrs. of total service on Machine #4

Let x_5 = Hrs. of total service on machine #5

Since there is no history prior to time x_o, it follows that x_1 , x_2 , x_3 , x_4 , and x_5 <u>are all greater than x_o</u> .

The ENTROPY of Machine #1 beyond x_o is

$$\mathcal{E}_1 = \ln \frac{1}{1 - F(x_1)} - \ln \frac{1}{1 - F(x_o)}$$

The ENTROPY on Machine #2 beyond x_o is

$$\mathcal{E}_2 = \ln \frac{1}{1 - F(x_2)} - \ln \frac{1}{1 - F(x_o)}$$

The ENTROPY on Machine #3 beyond x_o is

$$\mathcal{E}_3 = \ln \frac{1}{1 - F(x_3)} - \ln \frac{1}{1 - F(x_o)}$$

The ENTROPY on Machine #4 beyond x_o is

$$\mathcal{E}_4 = \ln \frac{1}{1 - F(x_4)} - \ln \frac{1}{1 - F(x_o)}$$

The ENTROPY on Machine #5 beyond x_o is

$$\mathcal{E}_5 = \ln \frac{1}{1 - F(x_5)} - \ln \frac{1}{1 - F(x_o)}$$

Then the TOTAL ENTROPY for the entire set of machines beyond x_o is

$$\boxed{\mathcal{E}_{\text{Total}} = \mathcal{E}_1 + \mathcal{E}_2 + \mathcal{E}_3 + \mathcal{E}_4 + \mathcal{E}_5} \tag{1}$$

Now , suppose that

Machine #1 has experienced r_1 failures since time x_o ,

Machine #2 has experienced r_2 failures since time x_o ,

Machine #3 has experienced r_3 failures since time x_o ,

Machine #4 has experienced r_4 failures since time x_o ,

Machine #5 has experienced r_5 failures since time x_o .

So, the TOTAL FAILURES experienced by the set of five machines since time x_o is

$$\boxed{r_{total} = r_1 + r_2 + r_3 + r_4 + r_5} \tag{2}$$

THE AVERAGE ENTROPY PER FAILURE SINCE THE HISTORICAL RECORD STARTING POINT x_o

The average entropy per failure since the initial time x_o is easily calculated by dividing (1) by (2) . Thus ,

$$\mathcal{E}_{ave.} = \frac{\mathcal{E}_{total}}{r_{total}} .$$

By ENTROPY THEORY , this $\mathcal{E}_{ave.}$ has the following <u>Normal</u> <u>Parameters:</u>

$$\text{MEAN} = 1$$

$$\text{STANDARD DEVIATION} = \frac{1}{\sqrt{r_{total}}}$$

CALCULATION OF THE CONFIDENCE INDEX

The NORMAL Z-SCORE for the calculated AVERAGE ENTROPY PER FAILURE since initial time x_o is

$$Z = \frac{\mathcal{E}_{ave.} - 1}{\frac{1}{\sqrt{r_{total}}}} = \sqrt{r_{total}}\left(\mathcal{E}_{ave.} - 1\right)$$

The CONFIDENCE of being AT LEAST AS GOOD AS THE RELIABILITY GOAL is then (in case $\mathcal{E}_{ave.} \geq 1$)

$$C = \text{Normal Area Out to } Z = \sqrt{r_{total}}\left(\mathcal{E}_{ave.} - 1\right)$$

$$= \mathcal{N}\left[\sqrt{r_{total}}\left(\mathcal{E}_{ave.} - 1\right)\right] .$$

In case $\mathcal{E}_{ave.} < 1$, then the Confidence C' of being INFERIOR TO THE GOAL is

$$C' = 1 - \mathcal{N}\left[\sqrt{r_{total}}\left(\mathcal{E}_{ave.} - 1\right)\right] .$$

A NUMERICAL EXAMPLE

Suppose the Reliability Goal for a certain machine model is

$$F(x) = 1 - e^{-\left(\frac{x}{4400}\right)^{1.2}}$$

This is a Weibull Cumulative Distribution of the two-parameter type with a Weibull Slope of 1.2 and a Characteristic Life of 4400 hours. From the basic definition of Entropy, this distribution has the Entropy formula

$$\mathcal{E} = \ln \frac{1}{1 - F(x)} = \left(\frac{x}{4400}\right)^{1.2}$$

Suppose that we have service history on five machines from 4000 hours and later, as follows :

MACHINE NO.	TOTAL HOURS OF SERVICE	FAILURES SINCE 4000 HRS.
1	4350	0
2	5000	1
3	6500	0
4	9000	0
5	12, 000	2
		(r_{total} = 3)

The five Entropies of these five machines beyond 4000 hours , within the goal F(x) as defined , are

$$\mathcal{E}_1 = \left(\frac{4350}{4400}\right)^{1.2} - \left(\frac{4000}{4400}\right)^{1.2} = .09445$$

$$\mathcal{E}_2 = \left(\frac{5000}{4400}\right)^{1.2} - \left(\frac{4000}{4400}\right)^{1.2} = .27386$$

$$\mathcal{E}_3 = \left(\frac{6500}{4400}\right)^{1.2} - \left(\frac{4000}{4400}\right)^{1.2} = .70525$$

$$\mathcal{E}_4 = \left(\frac{9000}{4400}\right)^{1.2} - \left(\frac{4000}{4400}\right)^{1.2} = 1.46827$$

$$\mathcal{E}_5 = \left(\frac{12,000}{4400}\right)^{1.2} - \left(\frac{4000}{4400}\right)^{1.2} = 2.44137$$

$$\left(\mathcal{E}_{total} \doteq 4.98320\right)$$

The AVERAGE ENTROPY PER FAILURE from the initial historical time x_o is then

$$\mathcal{E}_{ave.} = \frac{\mathcal{E}_{total}}{r_{total}} = \frac{4.98320}{3} = 1.66107$$

The NORMAL Z-SCORE for this $\mathcal{E}_{ave.}$ is

$$Z = \frac{1.66107 - 1}{\frac{1}{\sqrt{3}}} = \sqrt{3}\,(.66107) = 1.145$$

The CONFIDENCE of being AT LEAST AS GOOD as the Goal is then

$C = \mathcal{N}(1.145)$ = (Area under a Normal curve to 1.145 sigmas above the mean) = .87 = 87% Confidence (ANSWER)

CONCLUSION

The Average Entropy Per Failure Technique can be employed with any fixed historical starting time x_o, as long as we accumulate Entropies and Failures properly from that starting time x_o. The modification is very simple, and consists of subtracting $\ln[1/1 - F(x_o)]$ from each service time entropy on each machine under investigation. The accumulated entropy from x_o on is then the sum of such differences.

The MEAN of UNITY and the STANDARD DEVIATION of $1/\sqrt{\text{Total Failures}}$ are still valid parameters for $\mathcal{E}_{ave.}$ as accumulated beyond the initial time x_o, and for failures since time x_o.